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Stator / Rotor Slot Skewing (an MMF-harmonic-free Machine) and Tangential Harmonic Forces in Squirrel Cage Induction Machines; Analytic Calculation

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ABSTRACT

The skewing of the stator or the rotor is considered equivalent in literature. Since skewing the stator poses difficulties due to manufacturing technology, in practice, only the skewing of the rotor is applied. Therefore, the theory of stator skewing has not been developed. In this article, this gap will be filled. It will be demonstrated that the skewing of the stator or rotor is not equivalent; it is equivalent for a certain phenomenon, indeed, but not for another. Consequently, the effectiveness of rotor skewing alone cannot be complete; it is necessarily limited. Therefore, by applying the skewing of both the stator and rotor jointly in an appropriate manner, a complete result can be achieved: substantial attenuation of both stator and rotor MMF slot harmonics will be achieved. The second part of the article provides an analysis of the harmonic tangential forces. A formula is derived for the calculation of the harmonic torque waves acting on the tooth head of both stator and rotor (bringing them into bending vibration). With this result, the last missing formula is also available: we now have a concrete, numerical (not approximate) formula for all the effects of MMF harmonics in the cage asynchronous machine, as well as for the effect of slot skewing used to substantial attenuation each of them. Thus, the designer receives a complete and comprehensive picture and can make a well-founded decision regarding the application or rejection of a specific rotor slot number.

1. Introduction

Skewing is generally used by design engineers in the industry to mitigate the harmful effects of stator slot harmonics; therefore, the topic is discussed in every book dealing with electrical machines. However, all such investigations, which use an analytical method, are aimed only at showing and calculating the effect on the leakage reactance of the machine. This is stated to be independent of whether the stator slot or the rotor slot is skewed.

Nowadays, only numerical methods are applied for the investigation of this topic, and an analytical approach is currently lacking.

The goal of this paper is therefore to apply an analytical method to fill this gap and facilitate a better understanding.

In the following, first the true operating principle of rotor slot skew will be presented: a clear distinction is made between the effects on harmonic synchronous and asynchronous torques, the underlying physics is explained, and a new proposal is provided.

Then, the theory and the formulas available for calculating the harmonic tangential force will be studied because the actual, general situation seems not to be satisfactory.

Our paper is a direct continuation of the last publication [1]. The formulas of mine were developed in [2]. The new formulas were first implemented in [3]. The work is fundamentally based on [4], and the model serving as the basis for the calculations was taken from [5].

Nevertheless, for the current article to be understandable on its own, the necessary background will be repeated.

1.1. Analytical model

As a model, such a machine is assumed for which the basic formulas for determining the usual resulting space harmonics are valid [5]: infinite relative permeability, two-dimensional fields without considering boundary and end effects, the machine consists of two smooth coaxial cylinders made of magnetic material, the cylinders are separated by the air gap, and conductors of infinitely small

cross-section are located in the air gap.

In such a model, the distribution of the excitation current (no-load magnetizing current) flowing in the winding across the air gap will have the same shape as the magnetic induction or magnetic field, since there is a linear relationship between the two. Therefore, we often refer not to excitation, but to the air gap field and its harmonics. Considering its shape, the MMF curve is always "stepped"; consequently, it inherently contains harmonics.

Time harmonics and harmonics of other origins, as well as phenomena such as eccentricity and saturation etc., are excluded from this investigation not to distract the attention, to focus specifically on MMF space harmonics. The supply voltage is always considered sinusoidal, and the motor is started by connecting it directly to the network (without a frequency converter drive).

The well-known equivalent circuit of the asynchronous machine is valid only for the fundamental harmonic magnetic field; therefore, we have developed a comprehensive equivalent circuit that accounts for the full effects of harmonic fields for the model outlined above. So far, only harmonic asynchronous torques were included, now the equivalent circuit is *completed* by incorporating harmonic synchronous torques and harmonic magnetic forces as well.

The derivations as well as the conclusions drawn from the model are based on the fundamental laws of electrical engineering.

2. Symbols

R_1	stator ohmic resistance
X_{s1} - $X_{1\sigma}$	stator leakage without differential leakage
X_m	magnetizing reactance
$X_s = X_{s1} + X'_{s2}$	total leakage reactance
X'_{s2}	rotor total leakage reduced to stator
$X'_{2\sigma}$, $X'_{2\sigma\nu}$	rotor diff. leakage reduced to stator
R'_2, $R'_{2\nu}$	rotor resistance reduced to stator
ξ_1, ξ_ν	winding factor
$\xi_{1\nu_a}$, $\xi_{1\nu_b}$, ξ_1, ξ_ν	winding factor of harmonics, of fundamental harmonic
$\eta^2_{2\nu}$	Jordan's coupling factor
Δ	harmonic attenuation factor
s, s_ν	slip of rotor
X_m, X_s, X_{s1}, X_{s2}	reactance magnetizing / leakage, stator, rotor
ν, μ, ν', μ'	designation of stator, rotor harmonics
a, b	designation of harmonics in interaction
ν_a, ν_b, μ_a	harmonic order numbers stator, rotor
m, m_1, m_2	number of phases, stator, rotor
e, g	different integers: 0, ±1, ±2, ±3 …
f	maximum value of tension stress of a (sinus) radial, axial magnetic force wave [N/m^2]
$W_{magnetic}$	total magnetic energy of the machine
V_{airgap}, V_{rotor}	total volume of the air gap, of the rotor
p	number of pole pairs
Z_1, Z_2	slot number stator / rotor
r	order number of force wave
B, B_s, B_r, b_s, b_r	flux density
A_s, A_r, a_s, a_r	current layer
C	proportionality factor
I_1, I'_2, $I'_{2\nu}$	stator / (reduced) rotor current, harmonic current
x	location along the perimeter
δ'	equivalent air-gap
τ_p	pole pitch
l_i	effective (ideal) core length
N_1	turn per phase connected in series
μ_0	permeability of vacuum
θ, θ_m	MMF of one pole
M	torque
q_1, q'_2	relative slot numbers: stator/rotor slot number per pole per phase.
$q'_2 = q_2 + p/r$	where q_2 positive integer p, r positive integers and $p \le r$.
ω, ω_0, ω_s, ω_r	electrical angular velocity, of supply net, of stator /rotor harmonic
t	time (variable)
U	net voltage
σ_1, σ_2, $\sigma_{2\nu}$	differential leakage coefficient stator / rotor, rotor of ν^{th} asynchronous circuit

3. Results achieved so far

In the following, we will first reiterate those previously published findings on which the present results are based.

3.1. MMF harmonics in squirrel cage induction machines

MMF harmonics are given in first step for a 2p=2 pole machine.

Stator MMF harmonics

$$\nu = 2g \cdot m + 1 \tag{1}$$

where m stator phase number

g=0, ±1, ±2, ±3 …

the stator harmonics are independent of the stator slot number, they are always the same.

Rotor MMF harmonics

$$\mu_a = e\frac{Z_2}{p} + \nu_a \tag{2}$$

where ν_a the stator harmonic that created this rotor harmonic ($Z_2/p=m_2$ rotor phase number) the rotor harmonics are *dependent* on the rotor slot number; they are always distinct.

It is this latter fact that forms the basis of our approach. Therefore, we will *conduct our investigations, in contrast to the theoretical approach found in the literature* and standard industry practice, *"from the rotor's perspective."* Since any stator generates the same harmonics, searching for slot *combinations* starting from a given stator slot number is less effective—even though this is the standard design practice. Instead, we will examine what processes occur when a rotor with a given number of slots is installed in stators with different numbers of slots, consequently, stators with different slot harmonics. All our tables are based on this logic. As we will see, this is the only way to truly understand every process.

3.2. Completed equivalent circuit of squirrel cage induction machine

Known theorem: squirrel cage induction machine can be modeled with a harmonic-free basic machine in shaft-connection with infinite number of small asynchronous and synchronous machines [5]. Those small synchronous machines have, however, *never* been defined before.

Figure 1 shows, in addition to the harmonic-free main machine and small asynchronous machines, the small synchronous machines as well [1].

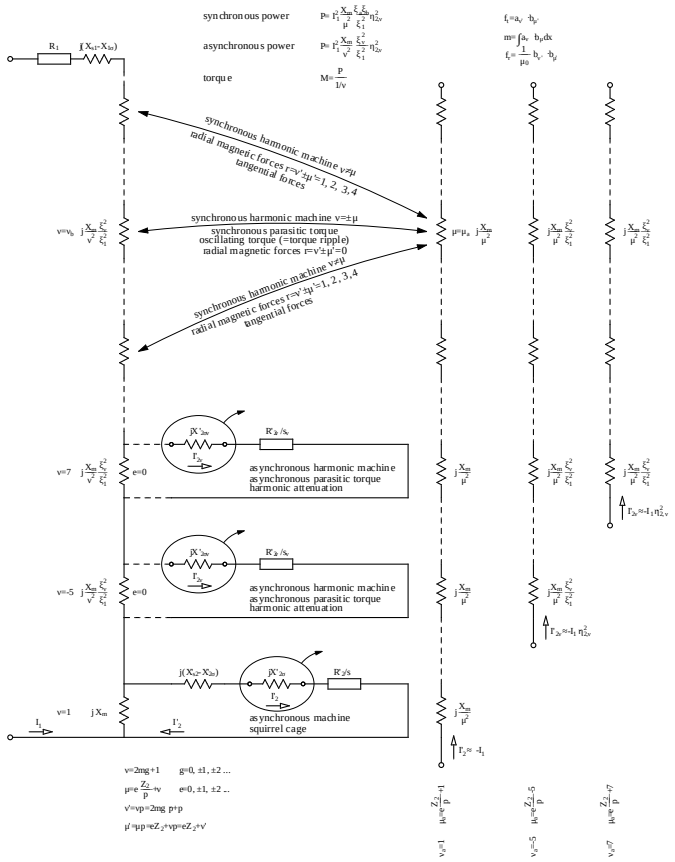


Figure 1. Single harmonic equivalent circuit of squirrel cage induction machine

In the figure, at the bottom, the known equivalent circuit represents the harmonic-free basic machine. The induction-machine circuits above it, starting from the stator MMF space harmonic (e.g. differential leakage) elements, represent the known small harmonic asynchronous machines. Then, rotor MMF space harmonics are also unfolded in the same columnar manner. The arrows connecting the stator and the rotor MMF space harmonic elements indicate the electromagnetic interaction between stator and rotor harmonic (e.g. differential leakage) elements. Because the elements, as well as the currents flowing through them, are independent of each other (that means neither is influenced nor induced by the other), the interaction is synchronous in nature, representing the small harmonic synchronous machines [1].

It is worth noting again that the stator and rotor harmonic elements of a harmonic synchronous machine are independent of each other; consequently, *both stator and rotor skewing only affect their "own" harmonics*, modifying only those that they themselves generate. *This is the physics on which the new arrangement suggested by this article relies.*

The figure shows that harmonic synchronous torque and harmonic radial magnetic forces are of the same origin: both are caused and thus explained by the same "small" harmonic synchronous machine. Therefore, the harmonic radial and tangential forces occurring in the small harmonic synchronous machines correspond directly to (and thus represent) the harmonic radial and tangential forces (incl. harmonic synchronous torques) occurring in squirrel-cage asynchronous machines.

3.2.1. The differential leakage reactance of the small harmonic asynchronous machines

The differential leakage reactance of the small harmonic asynchronous machines plays an important role in the behaviour of a squirrel cage machine:

$$X'_{2\sigma\nu} = \sigma_{2\nu} \cdot X_{m\nu} \tag{3}$$

where [4]

$$\sigma_{2\nu} = \frac{1}{\eta_{2\nu}^2} - 1 \qquad \text{rotor differential leakage coefficient}$$

where

$$\eta_{2\nu} = \frac{\sin \nu \dfrac{p\pi}{Z_2}}{\nu \dfrac{p\pi}{Z_2}} = \frac{\sin \nu \dfrac{\pi}{2mq'_2}}{\nu \dfrac{\pi}{2mq'_2}}$$

the so-called Jordan's coupling factor.

The factor shows the response of the cage on each stator harmonic. Figure 2 shows its characteristic curve as a function of the relative number of rotor slots q_2'.

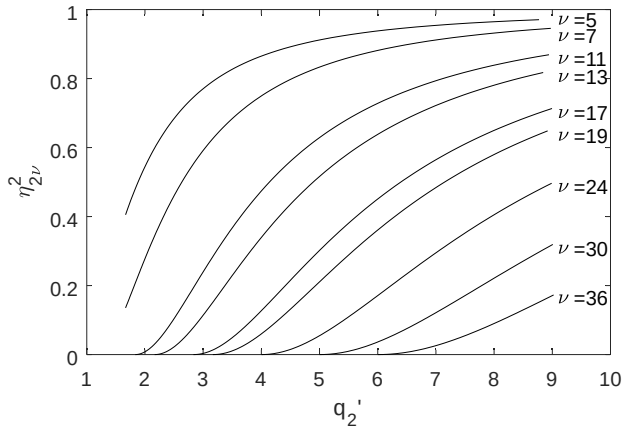


Figure 2. Representation of the value of $\eta_{2\nu}^2$ as a function of q_2' with ν as a parameter. Higher-order odd harmonic pairs are mathematically replaced by even harmonics between them to improve clarity [3]

3.3. The formulas we derived previously are used in the later calculations as follows

3.3.1. Harmonic synchronous (parasitic) torque

$$\frac{M_{synchronous}}{M_{breakdown}} = 2\frac{X_m}{X_s}\frac{\xi_{1\nu_a}\xi_{1\nu_b}}{\mu_a}\eta_{2\nu_a}^2\frac{1}{\xi_1^2} = 2\frac{X_m}{X_s}C \tag{4}$$

3.3.2. Harmonic radial magnetic forces

$$f_{rad} = \frac{1}{p\tau_p l_i \delta'} \cdot \left(\frac{m}{2} \cdot I_m^2 \frac{X_m}{2\pi f}\right) \cdot \left(\frac{\xi_{\nu_b}\xi_{\nu_a}}{\nu_b \mu_a}\eta_{2\nu_a}^2\frac{1}{\xi_1^2}\right) \tag{5}$$

$$f_{rad} = \frac{W_{magn}}{V_{airgap}/2} C \qquad C = \frac{\xi_{\nu_b}\xi_{\nu_a}}{\nu_b \mu_a} \eta^2_{2\nu_a} \frac{1}{\xi_1^2}$$

Noise Component Equivalence Measure

$$NCEM = 10\lg(10^4 C) = 10\lg\left(10^4 \frac{\xi_{1\nu_a}\xi_{1\nu_b}}{\nu_b \mu_a} \eta^2_{2\nu_a} \frac{1}{\xi_1^2}\right) \qquad (6)$$

3.3.3. Harmonic asynchronous (parasitic) torque

$$\frac{M_{b\nu}}{M_{break}} = \frac{X_m}{X_s} \frac{\xi_\nu^2}{\xi_1^2} \frac{\eta^2_{2\nu}}{\nu} \qquad (7)$$

3.3.4. Harmonic synchronous torques and harmonic radial magnetic forces in three-phase machines

Now and in each subsequent example, a machine with a relative rotor slot number of 2 1/3 is used, which would mean 28 rotor slots for a 4-pole machine. A machine with *typical characteristics* of $X_m/X_s = 15$ was taken as a basis with $X_m = 3$ p.u. and $X_s = 0.2$ p.u.; with these values, M_{break} is usually ≈ 2 p.u. Table 1. provides the resulting values of the machine with the used relative rotor slot number, calculated acc. to (4), (5) and (6). The oscillating torque (in rated operation) is the synchronous torque divided by the square of the ratio $I_{starting}/I_{rated} = 5$.

3.3.5. Design process of low-noise machines: evaluation of a rotor slot number

The process is presented for a three-phase, 4-pole machine with a rotor slot number $Z_2 = 28$.

The design process starts with Table 1, which shows the harmonic synchronous torques generated by the investigated rotor slot number to determine whether the machine will be able to start. The table also provides preliminary information by calculating the noise proportionality factor for the components created by the radial magnetic forces r=0 corresponding to the harmonic synchronous torques.

Then follows Table 2, which is essential for any noise investigation. Such a table is required for *each pole number separately;* therefore, in contrast to Table 1, which is independent of the pole number, each subsequent table (only because the noise components of the r>0 order are also now incorporated) refers to a single pole number only. Table 2, therefore, is provided now for 4-pole machines; the selected rotor slot number Z_2=28 and its integer multiples are highlighted.

The cell of the table in which the investigated slot number or its integer multiple appears identifies a radial force wave with the corresponding order number $\underline{r}$ and its associated frequency (for a 50 Hz power supply). As is well known, not only the harmonics ν_b appearing in the row of the actual q_1, but also those in the sections of integer multiples of q_1, represent slot harmonics.

The table has been supplemented with the μ_a harmonic produced by the investigated rotor slot number.

Then, the final task is to insert the additional force waves found in Table 2, identified with order r>0, into Table 1 to obtain Table 3. like a Noise Spectra caused by the selected rotor slot number. The symbol ≈0 in the column NCEM arising along with a negligible harmonic synchronous torque at standstill would substitute a very high negative value, which signifies a negligible noise component. Please note that harmonic synchronous torques at standstill created by non-slot-harmonics are negligible [4].

Table 1. Harmonics, harmonic synchronous torques, harmonic oscillating torques and harmonic radial magnetic force waves of a three-phase machine for a relative rotor slot number q_2'= 2 1/3.

m=		**3**	ν_b			q_1			ξ_b			Synchronous Torque / Breakdown Torque			Oscillating Torque / Breakdown Torque %			C proportionality factor for radial magnetic force wave density f [N/m²] *10000			**N**oise **C**omponent **E**quivalence **M**easure		
q_2	p/r	e	ν_b=-μ_a	ν_b=μ_a	operation	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4
2	**1/3**	-1	13		motoring	ξ_{slot}			-0,966	0,218	0,126	-2,23	0,50	0,29	-8,92	2,01	1,16	-57,16	12,87	7,46	17,57	11,10	8,73
		2	-29		breaking				0,259	-0,177	-0,205	-0,27	0,18	0,21	-1,07	0,73	0,85	3,08	-2,11	-2,44	4,88	3,24	3,88
		-3		-41	standstill				-0,259	0,218	-0,158	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0
		3		43					0,259	-0,177	0,205												
		-4	55		motoring		ξ_{slot}		-0,259	0,960	-0,158	-0,14	0,52	-0,09	-0,56	2,09	-0,34	-0,86	3,17	-0,52	-0,68	5,01	-2,83
		5	-71		breaking	ξ_{slot}	ξ_{slot}	ξ_{slot}	0,966	0,960	-0,958	-0,41	-0,41	0,40	-1,63	-1,62	1,62	1,92	1,90	-1,90	2,82	2,80	2,79
		-6		-83	standstill	ξ_{slot}			-0,966	-0,177	0,126	0,35	≈0	≈0	1,40	≈0	≈0	-1,40	≈0	≈0	1,47	≈0	≈0
		6		85					-0,966	0,218	-0,126	-0,34			-1,36			-1,34			1,26		
		-9		-125	standstill		ξ_{slot}		0,259	0,960	-0,205	≈0	-0,23	≈0	≈0	-0,92	≈0	≈0	0,61	≈0	≈0	-2,12	≈0
		9		127					-0,259	0,960	0,158		0,23			0,91			0,60			-2,25	
		-12		-167	standstill	ξ_{slot}		ξ_{slot}	0,966	0,218	-0,958	-0,17	≈0	0,17	-0,69	≈0	0,69	0,35	≈0	-0,34	-4,60	≈0	-4,64
		12		169					0,966	-0,177	-0,958	0,17		-0,17	0,69		-0,68	0,34		-0,34	-4,71		-4,75

Table 2. Order number and frequency (at 50 Hz) of harmonic radial magnetic force waves generated by different rotor slot numbers, in 3 phase, 4-pole machines; the just investigated Z_2=28 rotor slot number highlighted

				p=	2												
				r=	0			r=	1	r=	2	r=	3	r=	4		
				+	-			+	-	+	-	+	-	+	-		
q_1	Z_1	ν_b	ν'_b			μ_a	μ_a									μ_a	μ_a
2	24	-11	-22	20	24			21	25	22	26	23	27	24	28		-13
			freq.	600	600			625	625	650	650	675	675	700	700		
		-11	-22	20	24			19	23	18	22	17	21	16	20		
			freq.	600	600			575	575	550	550	525	525	500	500		
		13	26	28	24	-13		29	25	30	26	31	27	32	28		15
			freq.	600	600			625	625	650	650	675	675	700	700		
		13	26	28	24	-13		27	23	26	22	25	21	24	20		
			freq.	600	600			575	575	550	550	525	525	500	500		
3	36	-17	-34	32	36			33	37	34	38	35	39	36	40		
			freq.	900	900			925	925	950	950	975	975	1000	1000		
		-17	-34	32	36			31	35	30	34	29	33	28	32	15	
			freq.	900	900			875	875	850	850	825	825	800	800		
		19	38	40	36			41	37	42	38	43	39	44	40		
			freq.	900	900			925	925	950	950	975	975	1000	1000		
		19	38	40	36			39	35	38	34	37	33	36	32		
			freq.	900	900			875	875	850	850	825	825	800	800		
4	48	-23	-46	44	48			45	49	46	50	47	51	48	52		
			freq.	1200	1200			1225	1225	1250	1250	1275	1275	1300	1300		
		-23	-46	44	48			43	47	42	46	41	45	40	44		
			freq.	1200	1200			1175	1175	1150	1150	1125	1125	1100	1100		
		25	50	52	48			53	49	54	50	55	51	56	52	-27	
			freq.	1200	1200			1225	1225	1250	1250	1275	1275	1300	1300		
		25	50	52	48			51	47	50	46	49	45	48	44		
			freq.	1200	1200			1175	1175	1150	1150	1125	1125	1100	1100		
5	60	-29	-58	56	60	29		57	61	58	62	59	63	60	64		
			freq.	1500	1500			1525	1525	1550	1550	1575	1575	1600	1600		
		-29	-58	56	60	29		55	59	54	58	53	57	52	56		-27
			freq.	1500	1500			1475	1475	1450	1450	1425	1425	1400	1400		
		31	62	64	60			65	61	66	62	67	63	68	64	-33	33
			freq.	1500	1500			1525	1525	1550	1550	1575	1575	1600	1600		
		31	62	64	60	-31	31	63	59	62	58	61	57	60	56		29
			freq.	1500	1500			1475	1475	1450	1450	1425	1425	1400	1400		
7	84	-41	-82	80	84		-41	81	85	82	86	83	87	84	88	43	
			freq.	2100	2100			2125	2125	2150	2150	2175	2175	2200	2200		
		-41	-82	80	84		-41	79	83	78	82	77	81	76	80		
			freq.	2100	2100			2075	2075	2050	2050	2025	2025	2000	2000		
		43	86	88	84		43	89	85	90	86	91	87	92	88		
			freq.	2100	2100			2125	2125	2150	2150	2175	2175	2200	2200		
		43	86	88	84		43	87	83	86	82	85	81	84	80	-41	
			freq.	2100	2100			2075	2075	2050	2050	2025	2025	2000	2000		
9	108	-53	-106	104	108			105	109	106	110	107	111	108	112		-55
			freq.	2700	2700			2725	2725	2750	2750	2775	2775	2800	2800		
		-53	-106	104	108			103	107	102	106	101	105	100	104		
			freq.	2700	2700			2675	2675	2650	2650	2625	2625	2600	2600		
		55	110	112	108	-55		113	109	114	110	115	111	116	112		57
			freq.	2700	2700			2725	2725	2750	2750	2775	2775	2800	2800		
		55	110	112	108	-55	55	111	107	110	106	109	105	108	104		
			freq.	2700	2700			2675	2675	2650	2650	2625	2625	2600	2600		
10	120	-59	-118	116	120			117	121	118	122	119	123	120	124		
			freq.	3000	3000			3025	3025	3050	3050	3075	3075	3100	3100		
		-59	-118	116	120			115	119	114	118	113	117	112	116	57	
			freq.	3000	3000			2975	2975	2950	2950	2925	2925	2900	2900		
		61	122	124	120			125	121	126	122	127	123	128	124		
			freq.	3000	3000			3025	3025	3050	3050	3075	3075	3100	3100		
		61	122	124	120			123	119	122	118	121	117	120	116		
			freq.	3000	3000			2975	2975	2950	2950	2925	2925	2900	2900		
11	132	-65	-130	128	132			129	133	130	134	131	135	132	136		
			freq.	3300	3300			3325	3325	3350	3350	3375	3375	3400	3400		
		-65	-130	128	132			127	131	126	130	125	129	124	128		
			freq.	3300	3300			3275	3275	3250	3250	3225	3225	3200	3200		
		67	134	136	132			137	133	138	134	139	135	140	136	-69	
			freq.	3300	3300			3325	3325	3350	3350	3375	3375	3400	3400		
		67	134	136	132			135	131	134	130	133	129	132	128		
			freq.	3300	3300			3275	3275	3250	3250	3225	3225	3200	3200		
12	144	-71	-142	140	144	71		141	145	142	146	143	147	144	148		
			freq.	3600	3600			3625	3625	3650	3650	3675	3675	3700	3700		
		-71	-142	140	144	71		139	143	138	142	137	141	136	140		-69
			freq.	3600	3600			3575	3575	3550	3550	3525	3525	3500	3500		
		73	146	148	144			149	145	150	146	151	147	152	148		
			freq.	3600	3600			3625	3625	3650	3650	3675	3675	3700	3700		
		73	146	148	144			147	143	146	142	145	141	144	140		71
			freq.	3600	3600			3575	3575	3550	3550	3525	3525	3500	3500		
14	168	-83	-166	164	168		-83	165	169	166	170	167	171	168	172	85	
			freq.	4200	4200			4225	4225	4250	4250	4275	4275	4300	4300		
		-83	-166	164	168		-83	163	167	162	166	161	165	160	164		
			freq.	4200	4200			4175	4175	4150	4150	4125	4125	4100	4100		
		85	170	172	168		85	173	169	174	170	175	171	176	172		
			freq.	4200	4200			4225	4225	4250	4250	4275	4275	4300	4300		
		85	170	172	168		85	171	167	170	166	169	165	168	164	-83	
			freq.	4200	4200			4175	4175	4150	4150	4125	4125	4100	4100		

Table 3. Harmonics and their effects of a three-phase, q_2' = 2 1/3 4 pole machine, Z_2=28; Calculated Noise Spectra

m=	3					q_1			Synchronous Torque / Breakdown Torque			Noise Component Equivalence Measure			Freq.
q_2	+1/3			operation	order nr.	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	Hz
	e	ν_b	μ_a		r=										
2	-1	-11	-13		4	ξ_{slot}						18,30	10,94	9,45	700
	-1	13	-13	motoring	0	ξ_{slot}			-2,23	0,50	0,29	17,57	11,10	8,73	600
	1	13	15		4	ξ_{slot}						16,95	10,48	8,11	700
	1	-17	15		4		ξ_{slot}					10,06	15,76	7,91	800
	-2	25	-27		4	ξ_{slot}		ξ_{slot}				11,56	4,20	11,52	1300
	-2	-29	-27		4							5,19	3,55	4,19	1400
	2	-29	29	breaking	0				-0,27	0,18	0,21	4,88	3,24	3,88	1500
	2	31	29		4							4,59	3,84	2,44	1400
	-3	43	-41		4							1,67	0,03	0,66	2000
	-3	-41	-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	≈0
	3	43	43												
	3	-41	43		4							1,67	0,91	-0,49	2200
	-4	-53	-55		4		ξ_{slot}					-0,52	5,18	-1,52	2800
	-4	55	-55	motoring	0		ξ_{slot}		-0,14	0,52	-0,09	-0,68	5,01	-2,83	2700
	4	55	57		4		ξ_{slot}					-0,83	4,86	-2,99	2800
	4	-59	57		4	ξ_{slot}						4,58	-1,89	-4,26	1900
	-5	67	-69		4							-2,52	-3,27	-3,52	3400
	-5	-71	-69		4	ξ_{slot}	ξ_{slot}	ξ_{slot}				2,95	2,92	2,91	3500
	5	-71	71	breaking	0	ξ_{slot}	ξ_{slot}	ξ_{slot}	-0,41	-0,41	0,40	2,82	2,80	2,79	3600
	5	73	71		4	ξ_{slot}	ξ_{slot}	ξ_{slot}				2,70	2,68	2,67	3500
	-6	85	-83		4	ξ_{slot}						1,36	-5,11	-7,48	4100
	-6	-83	-83	standstill	0	ξ_{slot}			0,35	≈0	≈0	1,47	≈0	≈0	4200
	6	85	85			ξ_{slot}			-0,34			1,26			
	6	83	85		4	ξ_{slot}						1,36	-6,00	-7,48	4300
	-9	-125	-125	standstill	0		ξ_{slot}		≈0	-0,23	≈0	≈0	-2,12	≈0	6300
	9	127	127							0,23			-2,25		
	-12	-167	-167	standstill	0	ξ_{slot}		ξ_{slot}	-0,17	≈0	0,17	-4,60	≈0	-4,64	8400
	12	169	169						0,17		-0,17	-4,71		-4,75	

This table (without skewing) will later serve as the reference when evaluating different skewing arrangements.

4. Rotor slot skew

4.1. Critique of generally accepted explanation of the principle of effect of rotor slot skew

The operational effect is generally examined as shown in Figure 3. The explanation accompanying the figure states that it is advisable to skew the rotor slots by one stator slot pitch. In this case, the inducing influence of stator slot harmonics on the rotor bars is effectively eliminated.

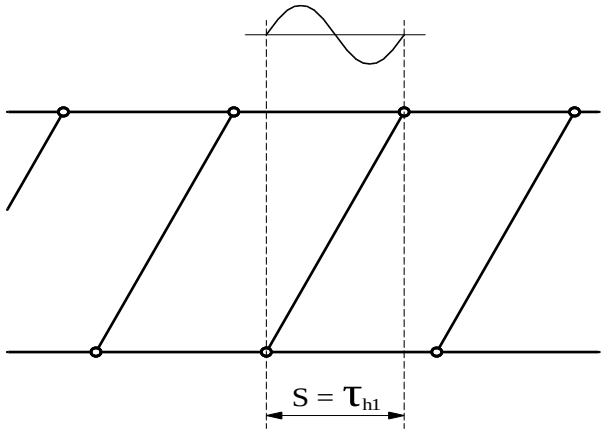


Figure 3. Sketch for explanation of principle of operation of rotor bar skew, s – skew, τ_{h1} – stator slot pitch

Figure 3 shows a configuration in which the bars of the rotor are skewed by exactly one stator slot pitch; since the $2mq_1$ (otherwise not occurring in reality) harmonic MMF of the stator along the length of the rotor bar completes one full period, no voltage is induced in it. In other words, the voltage induced in the rotor bar and its phase position, while travelling along the length of the bar, moves along a circle, which returns to the initial position upon reaching the end of the bar. The wavelengths of the adjacent harmonics, called $2mq_1 \pm 1$ slot harmonics, which occur, indeed, differ only slightly from the previous one, so they induce only a negligible voltage in the bar, that is, their effect on the machine's operation thus becomes negligible. Instead of the previous relationship $\xi_{slot} = \xi_1$, the *apparent* relationship $\xi_{slot} << \xi_1$ is established, the skew "removes" the stator slot harmonics.

This explanation is in two aspects incomplete and, therefore, misleading and incorrect.

Let us examine Figure 2. The figure's application is demonstrated for $q_1=3$ as an example: the corresponding stator slot harmonics are $\nu=17$ and $\nu=19$. If then $q_2' \leq 3$, the rotor does not respond to these or higher order harmonics, *regardless of whether the slots are skewed or not.*

Further, the explanation under critique has nothing to do with the generation of harmonic synchronous torque, as it has different physics.

4.2. Skewing by one stator slot pitch

4.2.1. General consideration

The (rotor) slot skew is – acc. to traditional derivation - usually modeled as if the number of slots on an arc equal to the pitch of a slot was infinite. As a matter of fact, Figure 4. ([6] Figure 113.) was originally developed for the winding distribution factor.

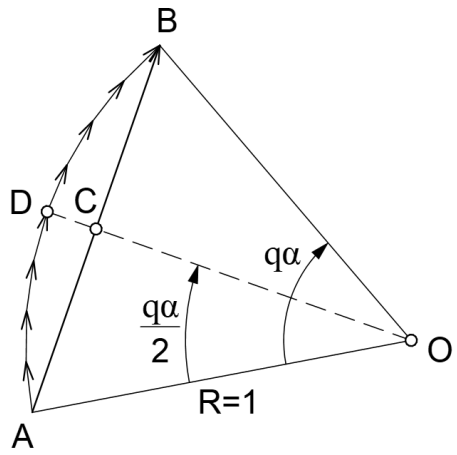


Figure 4. Calculation of the distribution winding factor for an infinite number of stator slots ([6] Figure 113.)

An infinite number of slots entails that the phase position of the voltage induced by the rotating main field in the winding-side sections that are infinitely close to each other moves along a circular arc. The qα range for the m=3 phase is π/3.

Thus, for a skew belonging to one slot pitch: $\alpha/2 = \pi/3 \cdot 1/2q_1 = \pi/6q_1$.

The skew factor acc. to this model (chord per arc) [6]

$$\xi_{skew} = \frac{\sin(\alpha/2)}{\alpha/2} = \frac{\sin(\pi/6q_1)}{\pi/6q_1} \tag{8}$$

for higher harmonics

$$\xi_{skew} = \frac{\sin(\nu\pi/6q_1)}{\nu\pi/6q_1} \tag{8a}$$

4.2.2. Consideration acc. to the effect of the skewing on the MMF

Previous approaches have focused primarily on the induced voltage, limiting themselves to the examination of that effect. Looking at Figure 4, one might think that this approach is, therefore, primarily valid for processes in the small harmonic *asynchronous* machines. However, it is evident from Figure 1 that this approach is incorrect when examining the effect on harmonic synchronous torques, as there is no induced voltage in this phenomenon. Because the stator and rotor of small harmonic synchronous machines are electromagnetically independent, their interaction exhibits synchronous machine characteristics. Therefore, the phenomenon should be investigated by analyzing the effect of skewing on the MMF, considering either stator or rotor slot skew.

Let us investigate the MMF curve of one phase of a 3-phase winding first for $q_1=1$ according to Figure 5. The non-skewed curve is represented by the solid line, while the trapezoidal MMF curve created as a result of the stator slot pitch skewing is represented by the dashed line.

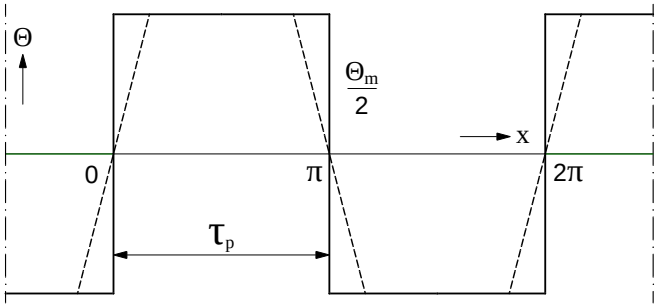


Figure 5. MMF curve of one phase of a three-phase winding, $q_1=1$.

Let us consider the harmonic content of both arrangements.

Fourier series of MMF created by two coil sides lying in non-skewed slots in a pole pitch distance as a function of location along the circumference see Figure 5. solid line.

$$\Theta(x) = \frac{4}{\pi}\frac{\Theta_m}{2}\left(\sin x + \frac{1}{3}\sin 3x + \ldots + \frac{1}{\nu_{slot}}\sin \nu_{slot} x + \ldots\right) \tag{9}$$

The harmonics of smallest order numbers acc. to (9) are shown in Figure 6.

Let us consider Figure 7. It is originally elaborated for infinite number of stator slots [6]. It is not difficult to notice that the slot skewing by one stator slot pitch and the infinite number of stator slots result in the identical MMF curve.

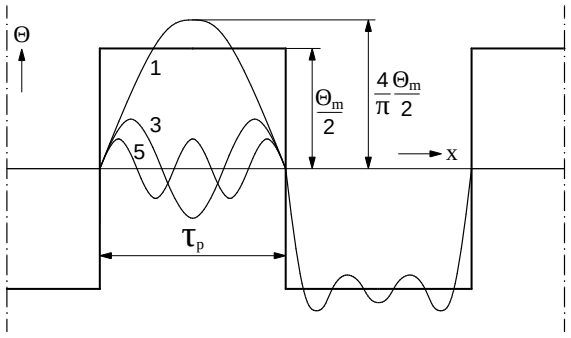


Figure 6. Harmonics of square MMF curve [6] Figure 151.

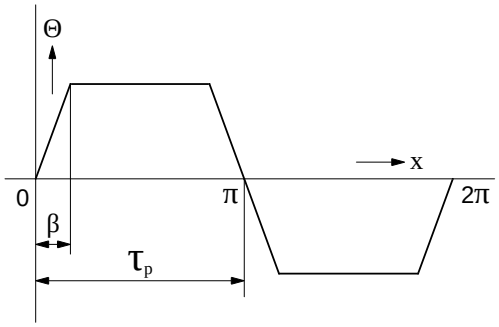


Figure 7. Trapezoidal MMF curve of infinite slot number (q=∞) ([6] Figure 152.)

The Fourier series of MMF created by two coil sides lying in skewed slots in a pole pitch distance as a function of location along the circumference (dashed line in Figure 5.) ([6] (99))

$$\Theta(x) = \frac{4}{\pi}\frac{\Theta_m}{2\beta}\left(\sin x \sin\beta + \frac{1}{3^2}\sin 3x \sin 3\beta + \ldots + \frac{1}{5^2}\sin 5x \sin 5\beta + \ldots\right) \quad (10)$$

For one phase of a three-phase winding, $\beta=\pi/3\cdot 1/2=\pi/6$, generally $\beta=\pi/2m_1$.

By the forming the ratio of (10) to (9) for each harmonic, all harmonics including the fundamental harmonic decrease due to skewing by the factor Δ_{skew} as follows, generally for any skewing

$$\Delta_{skew} = \frac{1}{\beta}\frac{1}{\nu}\sin(\nu\beta) = \frac{\sin\nu\beta}{\nu\beta} \quad (11)$$

Skewing by slot skew corresponding to any q_1 = integer, consequently $\beta=\pi/(2m_1q_1)$

$$\Delta_{skew} = \frac{\sin\nu\pi/6q_1}{\nu\pi/6q_1} \quad (12)$$

It should be mentioned here that, if the skewing is performed according to the rotor slot pitch, then $\beta=\pi/2m_1q_2'=\pi/2m_2$).

It is surprising that (8) and (12) give the same result, although the physical content is different. Formula (8) refers to the phenomenon where the skewing modifies the induced voltage in a bar. Formula (12) refers to the phenomenon where the skewing modifies the MMF created by the current flowing in a bar.

The explanation for the equivalence of these two expressions is given by a known *theorem*:

The higher order voltage harmonics induced in an arbitrary winding will be only those which the winding itself is able to produce by its own magnetic field intensity ([5] p. 31).

The physical phenomenon in (8) is associated with the induced voltage in the rotor bar, which relates to the creation of harmonic *asynchronous* torque. The physical phenomenon in (12) is associated with the MMF generated by the rotor bar, which relates to the creation of harmonic *synchronous* torque.

These statements clearly distinguish the physical difference between the effect of skewing on harmonic asynchronous and synchronous torques.

The known theorem above explains why slot skewing has been successfully applied ever since to reduce harmonic synchronous torques, though only on an intuitive basis.

4.2.3. Skewing of the stator slot by one stator slot pitch

Figure 5. refers to an arrangement with $q_1=1$ only. Therefore, although one might think the consideration may refer to stator slot skewing as well, (8), (11) and (12) can refer only to rotor slot skewing and cannot refer to a stator winding with $q_1>1$.

Stator with q_1 integer (full pitch) winding needs a different consideration, see Figure 8.

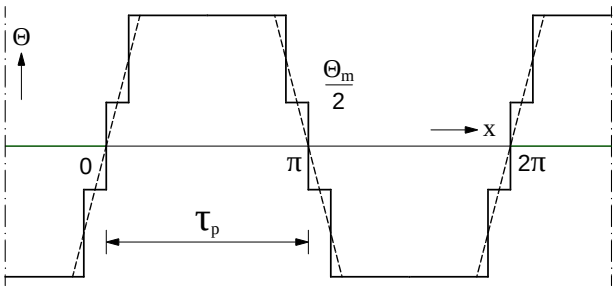


Figure 8. MMF curve of one phase of a three-phase winding, $q_1=3$, full pitch winding, in case of skewing acc. to one stator slot pitch

The skewed line of one complete phase is still identical to Figure 5, as if it were $q_1=1$: it is *independent* of the relative slot number q_1, but the magnitude of skew is only one-third ($1/q_1$ times) that of the previous one.

The formula of Fourier analysis of the solid line of Figure 8. is:

$$\Theta(x) = \frac{4}{\pi}\frac{\Theta_m}{2}\left(\xi_1 \sin x + \frac{1}{3}\xi_3 \sin 3x + \frac{1}{5}\xi_5 \sin 5x + \ldots + \frac{1}{\nu_{slot}}\xi_{\nu_{slot}} \sin \nu_{slot}x + \ldots\right) \quad (13)$$

where ξ_ν contains only the distribution factor, no chording, at the moment. The ratio has changed, since now the ratio of (13) to (9) (with $\beta=\pi/6$) shall be formed. The *basic* formula for stator MMF harmonic reduction factor due to stator slot skewing by exactly one stator slot pitch is:

$$\Delta_{stator} = \frac{1}{\xi_\nu}\frac{\sin\nu\beta}{\nu\beta} = \frac{1}{\xi_\nu}\frac{\sin\nu\cdot\pi/6}{\nu\cdot\pi/6} = \frac{1}{\xi_\nu}\frac{1}{\nu}\frac{3}{\pi} \quad (14)$$

again for ν=1, 5, 7, ...

In contrast to expressions (8) and (12), which include a trigonometric function, (14) leads to a very simple formula. The reason is that the usual stator winding generates -- in contrast to the rotor – always the same harmonics to be substituted in (14), and the value of the trigonometric function is the same: ±1/2, regardless of ν; which can thus be substituted in advance.

No formula exists for a stator slot skewing other than one stator slot pitch, nor does it have any practical importance; therefore, no other stator slot skewing configurations will be investigated.

4.2.4. Effect of skewing on harmonic (parasitic) torques

Effect on harmonic synchronous torque

$$\frac{M_{bv,skew}}{M_{break}} = \frac{M_{bv}}{M_{break}} \Delta_{skew} \tag{15}$$

Effect on harmonic asynchronous torque

$$\frac{M_{bv,skew}}{M_{break}} = \frac{M_{bv}}{M_{break}} \xi^2_{skew} \tag{16}$$

Although Δ_{skew} and ξ_{skew} are calculated by identical formula, equal symbols must not be used to indicate the clear difference in their physics.

5. Effect of rotor slot skewing on noise; the most effective rotor slot skewing

5.1. Considerations for determining the most appropriate rotor slot skewing

This time, however, instead of precise theoretical discussions and derivations, we find it more practical again to demonstrate the question and the solution with a numerical example; see Tables 4. The calculations were performed for skewing not only according to the rotor and the stator slot pitch but according to the critical harmonics as well. As expected, the effect of skewing is not uniform; it is different for each component, for each row in a table, meaning for each harmonic. It should be noted that the reduction factor shall be calculated according to μ_a: $\Delta_{rotor} = \sin\mu_a\beta/(\mu_a\beta)$ because the rotor is now skewed.

The tables dedicated again to q_2' = 2 1/3 and its column q_1 = 3 represents, for 4 poles, the slot number arrangement most commonly used in practice: Z_1/Z_2 = 36/28 slots.

Table 4a provides the values for the standard rotor slot skewing with one stator slot pitch.

The residual synchronous torque after skewing is constant if it is skewed by the *actual* stator slot pitch according to q_1 .

In cases where the lowest harmonic interactions are generated by non-stator slot harmonics, rotor slot skewing with one rotor slot pitch provides better results; Table 4b shows this for such skewing.

Based on the analysis of the tables above, alongside the rotor slot skewing referred to as basic skewing, there also emerge rotor slot skewing that can be described as ideal, namely skewing according to the harmonic order of the rotor involved in the interaction. One such possibility is shown in Table 4c.

The results of the other possibility of ideal skewing are shown in Table 4d.

Table 4a. Harmonics and their effects of a three-phase machine q_2' = 2 1/3 , 4 poles: Z_2=28 with rotor slot skewing *by one stator slot pitch: acc. to actual* q_1*:* $\beta=\pi/12$, $\beta=\pi/18$, $\beta=\pi/24$ *respectively*

m=	3					q1			Synchronous Torque / Breakdown Torque			Noise Component Equivalence Measure			Freq.
q2	+1/3			operation	order nr.	q1=2	q1=3	q1=4	q1=2	q1=3	q1=4	q1=2	q1=3	q1=4	Hz
	e	νb	μa		r=										
2	-1	-11	-13		4	ξslot						7,11	6,22	7,11	700
	-1	13	-13	motoring	0	ξslot			-0,17	0,17	0,17	6,38	6,38	6,38	600
	1	13	15		4	ξslot						9,50	3,29	4,83	700
	1	-17	15		4		ξslot					2,62	8,57	4,63	800
	-2	25	-27		4	ξslot		ξslot				1,56	-2,54	1,86	1300
	-2	-29	-27		4							-4,81	-3,18	-5,47	1400
	2	-29	29	breaking	0				-0,03	0,03	0,03	-4,07	-4,07	-4,07	1500
	2	31	29		4							-4,36	-3,47	-5,51	1400
	-3	43	-41		4							-8,79	-9,68	-7,64	2000
	-3	-41	-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	≈0
	3	43	43												
	3	-41	43		4							-25,37	-24,48	-26,52	2200

Table 4b. Harmonics and their effects of a three-phase, q_2' = 2 1/3 4 pole machine, Z_2=28 with rotor slot skewing *by one rotor slot pitch*: $\beta=\pi/14$

m=	3					q1			Synchronous Torque / Breakdown Torque			Noise Component Equivalence Measure			Freq.
q2	+1/3			operation	order nr.	q1=2	q1=3	q1=4	q1=2	q1=3	q1=4	q1=2	q1=3	q1=4	Hz
	e	νb	μa		r=										
2	-1	-11	-13		4	ξslot						7,12	-0,24	-1,72	700
	-1	13	-13	motoring	0	ξslot			-0,17	0,04	0,02	6,39	-0,08	-2,45	600
	1	13	15		4	ξslot						5,15	-1,32	-3,69	700
	1	-17	15		4		ξslot					-1,73	3,96	-3,89	800
	-2	25	-27		4	ξslot		ξslot				-2,79	-10,15	-2,83	1300
	-2	-29	-27		4							-9,16	-10,80	-10,16	1400
	2	-29	29	breaking	0				-0,01	0,01	0,01	-9,78	-11,42	-10,78	1500
	2	31	29		4							-10,07	-10,82	-12,22	1400
	-3	43	-41		4							-14,50	-16,14	-15,50	2000
	-3	-41	-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	≈0
	3	43	43												
	3	-41	43		4				0,00	0,00	0,00	-14,70	-15,46	-16,86	2200

Table 4c. Harmonics and their effects of a three-phase, q_2' = 2 1/3 4 pole machine, Z_2=28 with rotor slot skewing *acc. to critical harmonic* μ_a=13: $\beta=\pi/13$

m=	3					q1			Synchronous Torque / Breakdown Torque			Noise Component Equivalence Measure			Freq.
q2	+1/3			operation	order nr.	q1=2	q1=3	q1=4	q1=2	q1=3	q1=4	q1=2	q1=3	q1=4	Hz
	e	νb	μa		r=										
2	-1	-11	-13		4	ξslot						-141,60	-148,96	-150,45	700
	-1	13	-13	motoring	0	ξslot			0,00	0,00	0,00	-142,33	-148,80	-151,17	600
	1	13	15		4	ξslot						8,03	1,55	-0,81	700
	1	-17	15		4		ξslot					1,14	6,84	-1,01	800
	-2	25	-27		4	ξslot		ξslot				-2,80	-10,16	-2,84	1300
	-2	-29	-27		4							-9,16	-10,80	-10,17	1400
	2	-29	29	breaking	0				-0,03	0,02	0,02	-5,36	-7,00	-6,36	1500
	2	31	29		4							-5,65	-6,40	-7,80	1400
	-3	43	-41		4							-11,62	-13,26	-12,63	2000
	-3	-41	-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	≈0
	3	43	43												
	3	-41	43		4							-9,35	-10,10	-11,50	2200

Table 4d. Harmonics and their effects of a three-phase, q_2' = 2 1/3 4 pole machine, Z_2=28 with rotor slot skewing *acc. to critical harmonic* μ_a=15: $\beta=\pi/15$

m=	3					q1			Synchronous Torque / Breakdown Torque			Noise Component Equivalence Measure			Freq.
q2	+1/3			operation	order nr.	q1=2	q1=3	q1=4	q1=2	q1=3	q1=4	q1=2	q1=3	q1=4	Hz
	e	νb	μa		r=										
2	-1	-11	-13		4	ξslot						10,04	2,68	1,20	700
	-1	13	-13	motoring	0	ξslot			-0,33	0,08	0,04	9,31	2,84	0,47	600
	1	13	15		4	ξslot						-140,49	-146,96	-149,33	700
	1	-17	15		4		ξslot					-147,37	-141,68	-149,53	800
	-2	25	-27		4	ξslot		ξslot				1,72	-5,64	1,69	1300
	-2	-29	-27		4							-4,64	-6,28	-5,65	1400
	2	-29	29	breaking	0				-0,01	0,01	0,01	-9,77	-11,42	-10,78	1500
	2	31	29		4							-10,06	-10,82	-12,22	1400
	-3	43	-41		4							-8,96	-10,60	-9,97	2000
	-3	-41	-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	≈0
	3	43	43												
	3	-41	43		4							-11,78	-12,54	-13,94	2200

Both ideal choices reduce the specific values generated by them

to zero.

The practical (factory workshop) meaning of the skewing π/18 is the following: the magnitude of a skew π/18 corresponds to one stator slot pitch at q_1=3. Skewing according to a skew of π/13 or π/15 is 14/13 times stronger or 14/15 times less than a skew of one rotor slot pitch, respectively. A distance corresponding to π is one pole, or a pole pitch.

In Table 4c, the critical harmonic synchronous torque and the harmonic radial force wave generated by μ_a=13, and in Table 4d, both harmonic radial force waves generated by μ_a=15, are reduced to zero.

Therefore, to find the optimal (best balanced between conflicting requirements) skew, a comprehensive investigation based on Tables 4 must be conducted.

For the decision, the effect on both harmonic synchronous torques and noise must be examined. When using slot skew, far fewer harmonics need to be examined, because the skew inherently eliminates the effects of higher harmonics; as they appear in the denominator of the formulas (see (4) and (5)).

While a definite, general statement can be made regarding harmonic synchronous torque and even a theorem can be formulated, with respect to noise, however, considerations specific to the actual machine (e.g., natural frequencies) must be performed first; a well-founded decision regarding which noise component to eliminate can be made thereafter.

Regarding harmonic synchronous torque, therefore, the slot skew according to β=π/13 must be applied definitively and unambiguously; regarding noise, however, the slot skew according to β=π/15 will probably provide the most effective solution; the right decision, in the end, might be a compromise.

The harmonic asynchronous torque is not a matter for consideration. Here, it isn't considered at all because Z_2<Z_1. But generally, it is not considered either, because the harmonic asynchronous torque decreases with the square of the skew factor (16), so in the end, whatever skew arises, it will eliminate the harmonic asynchronous torque in any case.

Theorem:

A harmonic synchronous torque in rotation and the corresponding r=0 radial harmonic magnetic force component, generated by a rotor harmonic classified as critical, can be reduced to a mathematical zero value by rotor slot skewing corresponding to that harmonic.

A harmonic magnetic force wave component with an r≠0 order, generated by a rotor harmonic classified as critical, can be reduced to a mathematical zero value by rotor slot skewing corresponding to that harmonic.

A harmonic synchronous torque at standstill and the corresponding r=0 radial harmonic magnetic force component, generated by a rotor harmonic pair classified as critical, cannot be reduced to a mathematical zero value by slot skewing corresponding to that harmonic pair; they can only be reduced to a small, finite value.

It should be noted that in this widely used slot number arrangement, the exclusively applied β=π/18 (stator slot pitch) skew provides the least effective solution (not because it represents the least degree of skewing).

When calculating the effect of slot skew, the change in the leakage reactance is not considered for the moment.

5.2. Equivalence of stator and rotor slot skewing

The findings above have made it possible to continue the research in this direction.

We clarify here the general understanding regarding the equivalence of skewing on the stator versus the rotor. To demonstrate this, *stator* skewing is calculated for one stator slot pitch in Table 5; the reduction factor shall now be calculated according to ν_b: $\Delta_{stator}=\sin\nu_b\beta/(\nu_b\beta)$, as it is the stator which is now skewed.

The physical basis for calculating the reduction factor is that stator and rotor differential leakage components (Figure 1), considered as the "stator" and "rotor" components of a harmonic synchronous machine, are independent from each other; consequently, the mechanical intervention by stator or rotor skewing only affects their "own" harmonics; they only modify those that they themselves generated.

Table 5. Harmonics and their effects of a three-phase, q_2' = 2 1/3 4 pole machine, Z_2=28 with *stator* slot skewing *by one stator slot pitch: acc. to actual* q_1*:* $\beta=\pi/12$, $\beta=\pi/18$, $\beta=\pi/24$ *respectively*

m=	3					q_1			Synchronous Torque / Breakdown Torque			Noise Component Equivalence Measure			Freq.
q_2	+1/3			operation	order nr.	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	Hz
	e	ν_b	μ_a		r=										
2	-1	-11	-13		4	ξ_{slot}						7,83	7,83	7,83	700
	-1	13	-13	motoring	0	ξ_{slot}			-0,17	0,17	0,17	6,38	6,38	6,38	600
	1	13	15		4	ξ_{slot}						5,76	5,76	5,76	700
	1	-17	15		4		ξ_{slot}					3,43	3,43	3,43	800
	-2	25	-27		4	ξ_{slot}		ξ_{slot}				-2,47	-2,47	-2,47	1300
	-2	-29	-27		4							-3,76	-3,76	-3,76	1400
	2	-29	29	breaking	0				-0,03	0,03	0,03	-4,07	-4,07	-4,07	1500
	2	31	29		4							-4,65	-4,65	-4,65	1400
	-3	43	-41		4							-9,00	-9,00	-9,00	2000
	-3	-41	-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	≈0
	3	43	43												
	3	-41	43		4							-8,79	-8,79	-8,79	2200

Now, Table 4a and 5 shall be compared. It is obvious that stator skew does not modify harmonics 15, 27, etc., in the airgap because it has no influence on them; it only influences harmonics 11, 13, 17, 19, etc. Therefore, skewing the stator by one slot pitch would yield results different from Table 4a. Furthermore, it is interesting to note in Table 5 that not only the harmonic synchronous torques but also the NCEM values are independent of q_1. It makes no sense to analyze this fact further, as it lacks practical importance; however, a basic theorem can still be formulated.

Theorem:

The expression "equivalent" means that if there are any harmonics among those of the stator and rotor whose order numbers are identical, they are modified to the same extent. Regarding harmonic synchronous torques and r=0 order harmonic magnetic forces, therefore, skewing on either the stator or the rotor is equivalent.

For those harmonics of the stator and rotor that are not identical, skewing on the stator and rotor cannot be equivalent because each only modifies its "own" harmonics. Regarding harmonic radial magnetic force waves (creating noise) of r>0 order, therefore, skewing on the stator and rotor is not equivalent.

The result of this investigation regarding stator slot skew, which revealed that the equivalence of stator and rotor skew is not univer-

sal has led to the development of a new arrangement for squirrel-cage induction machines.

5.3. An MMF-harmonic-free machine

Up to this point, we have focused on rotor slot skewing, since this is the standard practice, and by studying it, we aimed to optimize the solution. Stator skewing was addressed only from the theoretical perspective of the equivalence of skewing, as it is not applied in practice due to manufacturing difficulties.

However, the physical law formulated above, stating that both sides, when skewed, reduces only their "own" harmonics puts the issue in a new light. It points to the *limitations* of rotor slot skewing, implying that *skewing the rotor* alone *can never provide a complete solution*; it may occur that a harmful stator harmonic may not be neutralized by rotor slot skewing.

Skewing the stator as well, however, may overcome this necessarily arising limitation, that means this further mechanical intervention may provide further possibilities for improvement.

In this new arrangement, stator slot skewing will substantially attenuate stator (slot) harmonics, while rotor slot skewing does the same with rotor (slot) harmonics. This new solution covers all harmonics and their effects.

In Table 6, this configuration is modeled.

Table 6. Harmonics and their effects of a three-phase, q_2' = 2 1/3 4 pole machine, Z_2=28 Skewing the stator by one *stator* slot pitch and skewing the rotor by one *rotor* slot pitch

m=	3					q_1			Synchronous Torque / Breakdown Torque			Noise Component Equivalence Measure			Freq.
q_2	+1/3			operation	order nr.	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	Hz
	e	ν_b	μ_a		r=										
2	-1	-11	-13		4	ξ_{slot}						-3,34	-3,34	-3,34	700
	-1	13	-13	motoring	0	ξ_{slot}			-0,01	0,01	0,01	-4,79	-4,79	-4,79	600
	1	13	15		4	ξ_{slot}						-6,04	-6,04	-6,04	700
	1	-17	15		4		ξ_{slot}					-8,37	-8,37	-8,37	800
	-2	25	-27		4	ξ_{slot}		ξ_{slot}				-16,82	-16,82	-16,82	1300
	-2	-29	-27		4							-18,11	-18,11	-18,11	1400
	2	-29	29	breaking	0				0,00	0,00	0,00	-18,73	-18,73	-18,73	1500
	2	31	29		4							-19,31	-19,31	-19,31	1400
	-3	43	-41		4							-25,16	-25,16	-25,16	2000
	-3	-41	-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	≈0
	3	43	43												
	3	-41	43		4							-25,16	-25,16	-25,16	2200

In each row, the value calculated without skewing in Table 3 was taken as a baseline and was multiplied by the factor calculated according to Δ_{stator}=sin$\nu_b\beta$/($\nu_b\beta$), where ν_b was taken from the ν_b column, *and then* multiplied *also* by the factor calculated according to Δ_{rotor}= sin$\mu_a\beta$/($\mu_a\beta$), where μ_a was taken from the μ_a column.

The result shall be compared, therefore, to Table 3 (without skewing) or to Table 4a (standard skewing), for the specially investigated slot number q_1 / q_2' = 3 / 2 1/3. In order to facilitate comparison, the results are summarized in one table, see Table 7.

where

Column I. represents the basic machine without skewing.

Column II. represents the values with standard skewing: rotor slot skewing acc. to one stator slot pitch.

Column III. represents the values with our proposal: rotor slot skewing acc. to one rotor slot pitch.

Column IV. represents the values with stator slot skewing acc. to one stator slot pitch.

Column V. represents the values with combined skewing: stator slot skewing by one stator slot pitch and rotor slot skewing by one rotor slot pitch.

Column VI. shows the difference, the reduction of the values by combined skewing compared to no skewing.

Once again: the results are *quantitative*.

Table 7. Harmonics and their effects of a three-phase, q_1 / q_2' = 3 / 2 1/3 4 pole machine : Z_1/Z_2=36/28, Skewing the stator by one *stator* slot pitch and skewing the rotor by one *rotor* slot pitch A comprehensive table to compare NCEM values achieved by different skewing

m=	3					q_1	Noise Component Equivalence Measure [dB]					
q_2	+1/3			operation	order nr.	q_1=3	q_1=3					
	e	ν_b	μ_a		r=		I	II	III	IV	V	VI
2	-1	-11	-13		4		10,94	6,22	-0,24	7,83	-3,34	14,28
	-1	13	-13	motoring	0		11,10	6,38	-0,08	6,38	-4,79	15,89
	1	13	15		4		10,48	3,29	-1,32	5,76	-6,04	16,51
	1	-17	15		4	ξ_{slot}	15,76	8,57	3,96	3,43	-8,37	24,12
	-2	25	-27		4		4,20	-2,54	-10,15	-2,47	-16,82	21,02
	-2	-29	-27		4		3,55	-3,18	-10,80	-3,76	-18,11	21,66
	2	-29	29	breaking	0		3,24	-4,07	-11,42	-4,07	-18,73	21,97
	2	31	29		4		3,84	-3,47	-10,82	-4,65	-19,31	23,15
	-3	43	-41		4		0,03	-9,68	-16,14	-9,00	-25,16	25,19
	-3	-41	-41	standstill	0		1,12	-8,58	-15,04	-8,58	-24,75	25,87
	3	43	43		0		-0,18	-9,20	-16,55	-9,20	-25,58	25,39
	3	-41	43		4		0,91	-8,11	-15,46	-8,79	-25,16	26,08

The difference is significant: the result is a machine that is (although not mathematically but practically) free of MMF harmonic torques and free of noise of magnetic origin. Please note that the result can not be mathematically free of harmonics because the skewing by any *full* slot pitch would reduce only the corresponding even harmonic to zero but stator harmonics are odd, rotor harmonics are (not always but in the most cases) also odd.

The rule regarding this combined skewing is the simplest possible; the designer can directly apply it without performing any preliminary harmonic analysis (as required for Table 3 and Tables 4) that was previously indispensable.

There may be applications where the advantage of being virtually free of harmonic magnetic noise, harmonic oscillating torque (=torque ripple) and force waves, and harmonic synchronous torques outweighs the difficulties associated with manufacturing the stator.

The solution gives the designers more freedom because they no longer need to strictly adhere to the so-called slot number rules and can focus their attention on other important fields, like selecting the rotor slot number to minimize additional losses.

6. Harmonic tangential magnetic forces

The harmonic tangential magnetic forces are discussed here on a comprehensive manner. Table 3 used so far is supplemented accordingly obtaining Table 9 (at the end of this chapter).

For a better demonstration of the physical processes, the system of the forces created by the action of the MMF harmonics is included in Table 8. The symbols and the approach in this chapter are based on [7].

Let us see the physical background of the relationship between tangential forces (including synchronous torques) and radial forces.

Both the flux density waves b_s – b_r and the current layer waves a_s – a_r of the stator and rotor resp., as a function of time and location on the periphery are written as follows:

$$\begin{aligned} b_s &= B_s \sin(\omega_s t - p_s x) \quad b_r = B_r \sin(\omega_r t - p_r x - \varphi) \\ a_s &= A_s \cos(\omega_s t - p_s x) \quad a_r = A_r \cos(\omega_r t - p_r x - \varphi) \end{aligned} \tag{17}$$

where ω_s, ω_r angular velocities of the fields
p_s, p_r pole pair number of the fields
$p_s = p\nu_b, \quad p_r = p\mu_a$

There is the only stipulation that there is a causality relationship between the quantities within the stator as well as within the rotor:

$$a = const\frac{db}{dx} \qquad \text{or} \qquad b = const\int a\,dx \tag{18}$$

There is no causality between stator and rotor quantities; both stator and rotor quantities can be of any origin, between them there is no forced relationship: this means that the whole phenomena will be of synchronous machine in nature.

When examining electrical machines, it is appropriate to look for the conditions for the generation of a constant torque in time.

However, for the purposes of researching this topic, it will be examined what happens when the conditions are not met because every possible combination occurs during the interaction between the harmonic waves of the stator and the rotor.

During the examination, we must allow positive or negative value not only for the angular velocity of the fields (the stator field relative to an assumed positive direction, the rotor field relative to the rotor, and the mechanical angular velocity of the rotor also relative to the same positive direction), but also both positive and negative values for the pole numbers, the latter showing the direction of rotation of the respective rotating harmonic field relative to the assumed positive direction.

A well-known formula regarding the relationship between radial and tangential forces

$$f_{tang} = \frac{p_r\delta}{R} f_{rad} = \frac{p\mu_a\delta}{R} f_{rad} \tag{19}$$

where δ the air gap and R the radius of the rotor

This formula is unlikely to be suitable for our requirements, as two geometric parameters appear in the equation, and their ratio also varies from machine to machine. The formula can be found in several references, e.g., [8], [9], dealing with noise only to demonstrate that, along with the radial forces causing noise, tangential forces also appear, and that these are negligible.

We cannot agree with this approach. Although harmonic radial forces are much greater, indeed, it is the yoke with its considerably larger mass, that is being set in motion (vibration), whereas the harmonic tangential forces cause the much lightweight tooth-and-coil-side arrangement to vibrate, specifically through the bending of the teeth; thus, individual teeth can be regarded as relatively independent regarding bending forces, and the mechanical connection between them is looser in any winding technology than in a ring (yoke) necessarily made (except for the large size) from a single piece of material. The danger of harmonic tangential forces, even if not decisive, cannot be ignored. On the other hand, this approach completely distracts from the fact that it is the harmonic tangential force which generates the harmonic synchronous and oscillating torques. This is why we consider Table 8 fundamentally important, as it presents the entire phenomenon in its interrelated context.

The formula itself appears to be correct, as it was successfully applied in [10] Part II, Section IV.B.2, while verifying our formulas for harmonic synchronous torque and radial magnetic force against each other (deriving one from the other using (19) for a force wave of order r=0). Nevertheless, in the cited derivation mentioned above, a *resultant* tangential force was actually obtained, defined as the *product* of the local tangential force and the perimeter. The harmonic tangential force wave, as a *wave* in space, only occurs for r≠0 (i.e., $p\nu_b \neq p\mu_a$). Existing derivations incl. (19) are, therefore, somewhat contradictory.

Let us recall the derivation in [7], which, relying on the fundamentals of the theory, applies to a completely general machine: p can be the pole pair number of the main machine, but it can also be the pole pair number $p\mu_a$ or $p\nu_b$ of a harmonic machine.

$$b = B\cos(px - \omega t) \tag{20}$$

Table 8. Radial and tangential harmonic magnetic forces (and torques) created in a squirrel cage induction machine

	Tangential force – Torque	Radial force
Basic formulas	$f_{tang} = B_s A_r \sin(\omega_s t - p_s x) \cdot \cos(\omega_r t - p_r x - \varphi) =$ $= 1/2 \cdot B_s A_r[\sin((\omega_s - \omega_r)t - (p_s - p_r)x + \varphi) +$ $+ \sin((\omega_s + \omega_r)t - (p_s + p_r)x - \varphi)]$	$f_{rad} \approx (B_s + B_r)^2 = B_s^2 + B_r^2 + 2B_sB_r =$ $= B_s^2/2 \cdot (1 - \cos(2\omega_s t - 2p_s x)) +$ $+ B_r^2/2 \cdot (1 - \cos(2\omega_r t - 2p_r x - 2\varphi)) +$ $+ 2B_sB_r \cdot 1/2 \cdot [\cos((\omega_s - \omega_r)t - (p_s - p_r)x + \varphi) -$ $- \cos((\omega_s + \omega_r)t - (p_s + p_r)x - \varphi)]$
$p_s \neq p_r$ $\omega_s \neq \omega_r$	$f_{tang} = 1/2 \cdot B_s A_r[\sin((\omega_s - \omega_r)t - rx + \varphi)$ $+ \sin((\omega_s + \omega_r)t - (p_s + p_r)x - \varphi)]$	$f_{rad} \approx \ldots + B_sB_r[\cos((\omega_s - \omega_r)t - rx + \varphi)$ $- \cos((\omega_s + \omega_r)t - (p_s + p_r)x - \varphi)]$
	f_{tang} tangential force waves along the periphery as function of the location on the periphery and the time $M = -lr^2 \int_0^{2\pi} f\,dx = 0$	Force of order r≠0

$$a = \frac{\delta}{R\mu_0}\frac{dB}{dx} = -\frac{p\delta}{R\mu_0}B\sin(px-\omega t) = -A\sin(px-\omega t) \quad (21)$$

where $A = \frac{p\delta}{R\cdot\mu_0}B$

The calculation in the opposite direction

$$b = \frac{\mu_0}{\delta}\int aR\,dx = A\frac{R\mu_0}{p\delta}\cos(px-\omega t) = \\ = \ldots = B\cos(px-\omega t) \quad (22)$$

Absolute value of the tangential force

$$f_{tang} = \frac{1}{2}\frac{p\delta}{R\cdot\mu_0}B_rB_s \quad (23)$$

Absolute value of the radial force

$$f_{rad} = \frac{1}{2\mu_0}B^2 = \ldots = \frac{1}{2\mu_0}B_sB_r \quad (24)$$

It follows

$$f_{tang}/f_{rad} = \frac{p\delta}{R} \quad (25)$$

(25) shows no more than how (19) was derived.

The torque of the machine, integrated along the entire perimeter.

$$M = const\cdot\int_0^{2\pi} ba\,dx = const\cdot B_sA_r\int_0^{2\pi}\ldots \quad (26)$$

Let us consider the physics of the phenomenon especially the relation of A to B in (21) that means the relationship between the radial magnetic field and the *local* circumferential current layer for the case of p=1. This relationship does not change if the number of pole pairs is p>1 instead of p=1. It is simply a matter of having further p-1 pole pairs arranged along the periphery of the machine the effects of which inlc. torque add up. Thus, in (21), the circumferential current layer A is *not* the real local layer, but a substitute value, which thereby correctly represents the total torque of the machine; however, it does not represent correctly the local physical characteristic, specifically the circumferential current layer. Therefore, A in (21) is p times the real local value of the current layer. Because of this, the correct procedure is to calculate with p = 1 in (20) through (23) and (25), and then to substitute this result into (26) as follows:

$$A(p=1) = \frac{\delta}{R\cdot\mu_0}B \quad (27)$$

$$M = const\cdot\int_0^{p\cdot 2\pi} ba\,dx = const\cdot p\int_0^{2\pi} B_sA_r(p=1)\ldots \quad (28)$$

Relation of the forces, therefore, correctly:

$$f_{tang} = \frac{\delta}{R}f_{rad} \quad (29)$$

Consequently the generally accepted (19) is incorrect.

Substituting these values into (5)

$$f_{tang} = \frac{1}{(p\tau_pl_iR)}\cdot\left(\frac{m}{2}\cdot I_m^2\frac{X_m}{2\pi f}\right)\cdot \\ \cdot\left(\frac{\xi_{\nu_b}\xi_{\nu_a}}{\nu_b\mu_a}\eta_{2\nu_a}^2\frac{1}{\xi_1^2}\right) = \frac{W_{magn}}{V_{rotor}}C \quad (30)$$

We have managed to derive a general formula once again. However, we do not recommend this approach going forward: the physics of tangential force is more related to that of harmonic synchronous torque than to that of harmonic radial force; furthermore, by following the physics of the former phenomenon, we will obtain a result that can be incorporated into our previous investigations (see Table 9 at the end of the chapter).

According to Figure 1, both the stator and the rotor of the modeled small harmonic synchronous machines have a forced-current arrangement in nature. Therefore, it is correct to start from the *harmonic current layer* of both stator and rotor, and determine the harmonic induction (flux density) of one side by integration, and not the opposite way - not from the derivation of the induction, as all literature does like [4], [7], [8], [9] - even though the result must be the same. During the derivation, we assume that the harmonic order numbers are close to each other, so they can (approximately) be used interchangeably.

The harmonic current layer labeled “b”

$$a_{\nu_b} = \frac{\sqrt{2}m}{p\tau_p}N_1\xi_{\nu_b}I_1\cos\left(\omega t - \frac{\nu_b\pi}{\tau_p}x\right) \quad (31)$$

The harmonic induction labeled “a”

$$b_{\mu_a} = \frac{\mu_0}{\delta'}\int a_{\mu_a}dx = \frac{\mu_0}{\delta'}\frac{\sqrt{2}m}{p\tau_p}\frac{\tau_p}{\mu_a\pi}N_1I_1\eta_{2\nu_a}^2\xi_{\nu_a}\cdot \\ \cdot\sin\left((1+(\mu_a-\nu_a)(1-s))\omega t - \frac{\mu_a\pi}{\tau_p}x+\ldots\right) \quad (32)$$

Substituting, the local harmonic tangential force

$$a_{\nu_b}b_{\mu_a} = \frac{\sqrt{2}m}{p\tau_p}\frac{\mu_0}{\delta'}\frac{\sqrt{2}m}{p\tau_p}\frac{\tau_p}{\mu_a\pi}N_1^2I_1^2\eta_{2\nu_a}^2\xi_{\nu_b}\xi_{\nu_a}\cdot \\ \cdot\sin(\ldots)\cos(\ldots) = \\ = \frac{\sqrt{2}m}{p\tau_p}\frac{\mu_0}{\delta'}\frac{\sqrt{2}m}{p\tau_p}\frac{\tau_p}{\mu_a\pi}N_1^2I_1^2\eta_{2\nu_a}^2\xi_{\nu_b}\xi_{\nu_a}\cdot \\ \cdot\frac{1}{2}\sin((\omega_s-\omega_r)t-rx)+\ldots \quad (33)$$

In (33), it is μ_a that appears in the formula. Should we start from $a_{\nu b}$ when deriving its induction, then ν_b would appear. According to physics, the product must be identical, it cannot depend on the starting point. If the calculation is used for harmonic synchronous torque where $\nu_b=\pm\mu_a$ it is not a matter. Here, however, where $\nu_b\neq\mu_a$ per definition, that requirement must be met. The precise calculation is laborious, so as an approximation, using the geometric mean of the two harmonics $\sqrt{\nu_b\mu_a}$ is suggested.

The local harmonic current is $A_{\nu b}\,dx$. Taken into account the formula in Table 8. for the $\underline{r}$ order of forces only, the local torque

acting in the air-gap is the local (Lorentz) tangential force multiplied by the radius of the rotor:

$$dm_{\nu_b\mu_a} = \frac{p\tau_p}{\pi} l_i \cdot A_{\nu_b} dx \cdot B_{\mu_a} \cdot \cdot \sin\left((\omega_s - \omega_r)t - r\frac{\pi}{p\tau_p}x + \varphi\right)\dots \quad (34)$$

The integral carried out over a specific portion of the perimeter provides the torque exerted by that part. The integral carried out over the full perimeter $2p\tau_p$ yields zero since the positive and negative half-waves of the r>0 order forces cancel each other. This corresponds to the fact that no torque is experienced on the shaft. The integral carried out only over a half-force-wave of r order, however, differs from zero, it provides the torque exerted by that half-wave *in the air-gap*, and therefore, this is now the appropriate way for calculation. This torque acts as a bending, vibrating force on the teeth; the value of the integral as known from the analysis: 2. The calculation procedure is acc. to [4] p. 191

$$m_{\nu_b\mu_a} = \frac{p\tau_p l_i}{\pi}\int a_{\nu_b} b_{\mu_a} dx = \\ = \frac{p\tau_p l_i}{\pi}\frac{2m^2}{p^2\tau_p^2}\frac{\mu_0}{\delta'}\frac{\tau_p}{\sqrt{\nu_b\mu_a}\pi} N^2 I^2 \eta_{2\nu_a}^2 \xi_{\nu_a}\xi_{\nu_b}\frac{1}{2}\cdot \\ \cdot \int_0^{\frac{2p\tau_p}{2r}} \sin\left((\omega_s - \omega_r)t - r\frac{\pi}{p\tau_p}x + \varphi\right)dx \quad (35)$$

$$m_{\nu_b\mu_a} = \frac{p\tau_p l_i}{\pi}\frac{2m^2}{p^2\tau_p^2}\frac{\mu_0}{\delta'}\frac{\tau_p}{\sqrt{\nu_b\mu_a}\pi}\cdot \\ \cdot N^2 I^2 \eta_{2\nu_a}^2 \xi_{\nu_a}\xi_{\nu_b}\frac{p\tau_p}{r\pi}\frac{1}{2}2$$

Arranging, substituting and then relating the result as usual [2] on the breakdown torque of the fundamental harmonic basic machine

$$M_{breakdown} = \frac{p}{2\pi f}\frac{mU^2}{2X_s}, \qquad \text{and} \qquad I \approx U/X_s \quad (36)$$

the result is

$$\frac{M_{tang}}{M_{breakdown}} = 2\frac{X_m}{X_s}\frac{1}{\pi r}\left(\frac{\xi_{\nu_a}\xi_{\nu_b}}{\sqrt{\nu_b\mu_a}}\eta_{2\nu_a}^2\frac{1}{\xi_1^2}\right) \quad (37)$$

$$\frac{M_{tang}}{M_{breakdown}} = \frac{M_{synchr}}{M_{break}}\frac{1}{r\pi} = \frac{M_{synchr}}{M_{break}}\frac{1}{2r}\frac{2}{\pi} \quad (37a)$$

The physical content of the formula is as follows: consider an imaginary synchronous machine created by harmonics of order $\sqrt{\nu_b\mu_a}$ that is by identical pole pair number that can generate a corresponding harmonic synchronous torque, indeed. Then, a 1/2r times of the total circumference of the air gap exerts 1/2r times of its total synchronous torque, which is thus identical to what the full circumference would exert. However, this portion produces only $2/\pi$ of that value, as it is not constantly distributed along the circumference but is sinusoidal; nevertheless, *its peak value* (at a total of 2r points symmetrical along the circumference) remains identical in magnitude.

Based on the considerations above, the final formula for the torque acting in the air-gap, sticking only to the essentials:

$$\frac{M_{tang}}{M_{breakdown}} = 2\frac{X_m}{X_s}\left(\frac{\xi_{\nu_a}\xi_{\nu_b}}{\sqrt{\nu_b\mu_a}}\eta_{2\nu_a}^2\frac{1}{\xi_1^2}\right)\cdot \\ \cdot \sin((\omega_s - \omega_r)t - rx) = \\ = 2\frac{X_m}{X_s}C\cdot\sin((\omega_s - \omega_r)t - rx) \quad (38)$$

Therefore, in the empty cells of Table 3 where r>0 and harmonic synchronous torque does not appear alongside harmonic radial magnetic forces, *travelling harmonic tangential force* waves—*with peak values corresponding to the geometric mean of the participating harmonics—and their associated harmonic torque* waves *still occur, both during startup and in operation.*

The up to now empty cells are filled with these peak values to obtain Table 9.

This resulting table comprehensively covers all aspects of the machine's operation in connection with MMF harmonics, particularly regarding noise.

It is reiterated that no machine geometric data are necessary, the results are still *quantitative*.

Table 9. Harmonics and their effects of a three-phase, q_2' = 2 1/3 4 pole machine, Z_2=28; Calculated Noise Spectra: Harmonic synchronous torques and oscillating torques for waves r>0 are not traditional synchronous torques acting on the shaft but sinusoidal distributed (synchronous) torque *waves* traveling along the circumference, which act only in the air gap

m=	**3**					q_1			Synchronous Torque / Breakdown Torque			Oscillating Torque / Breakdown Torque %			**N**oise **C**omponent **E**quivalence **M**easure			Freq.
q_2	**+1/3**			operation	order nr.	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	q_1=2	q_1=3	q_1=4	Hz
	e	ν_b	μ_a		r=													
2	-1	-11	-13		4	ξ_{slot}			-2,42	-0,44	-0,32	-9,69	-1,78	-1,27	18,30	10,94	9,45	700
	-1	13	-13	motoring	0	ξ_{slot}			-2,23	0,50	0,29	-8,92	2,01	1,16	17,57	11,10	8,73	600
	1	13	15		4	ξ_{slot}			-2,08	0,47	0,27	-8,30	1,87	1,08	16,95	10,48	8,11	700
	1	-17	15		4		ξ_{slot}		-0,49	1,80	0,30	-1,94	7,21	1,18	10,06	15,76	7,91	800
	-2	25	-27		4	ξ_{slot}		ξ_{slot}	1,12	-0,20	-1,11	4,46	-0,82	-4,42	11,56	4,20	11,52	1300
	-2	-29	-27		4				0,28	-0,19	-0,22	1,11	-0,76	-0,88	5,19	3,55	4,19	1400
	2	-29	29	breaking	0				-0,27	0,18	0,21	-1,07	0,73	0,85	4,88	3,24	3,88	1500
	2	31	29		4				-0,26	0,22	0,16	-1,04	0,87	0,63	4,59	3,84	2,44	1400
	-3	43	-41		4				0,18	-0,13	0,15	0,74	-0,51	0,59	1,67	0,03	0,66	2000
	-3	-41	-41	standstill	0				≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0	≈0
	3	43	43															
	3	-41	43		4				-0,18	0,16	-0,11	-0,74	0,62	-0,45	1,67	0,91	-0,49	2200

7. Summary

Building on our previous results and primarily utilizing the formulas we have previously derived, we continued our investigations to cover the entire area of slot skewing; that is, we also included stator slot skew in our studies. We investigated the equivalence of stator and rotor skewing and found that they are not equivalent. Therefore, a really satisfactory result cannot be achieved solely by skewing the rotor slots. In other words, skewing the stator as well may provide further possibilities for improvement. This consideration led to our current results, in which skewing is implemented on both sides. With this proposal, we have reached the limit: the examined properties of the machine cannot be further improved by slot skewing. The proposed solution is the simplest possible from the perspectives of both design and manufacturing at the same time. The solution gives designers more freedom because they no longer need to strictly adhere to the so-called slot number rules and can focus their attention on other important areas, such as selecting the rotor slot number to minimize additional losses. The second part of the article provides a further developed analysis of the harmonic tangential forces. A correct formula is derived for their calculation, and the table supplemented by the present result comprehensively covers all aspects of the machine's operation in connection with MMF harmonics, particularly regarding noise. A frequently used formula was proved by us to be incorrect.

Conflict of Interest The author declares no conflict of interest.

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