ASTES

Advances in Science, Technology and Engineering Systems Journal
Vol. 5, No. 4, 70-80 (2020)
www.astesj.com

ASTES Journal
ISSN: 2415-6698

Robust Static Output-Feedback Fault Tolerant Control for a Class of T-S Fuzzy Systems using Adaptive Sliding Mode Observer Approach

Slim Dhahri [*1,2], Essia Ben Alaia[1,3]

[1]*Department of Computer Engineering and Networks, College of Computer and Information Sciences, Jouf University, 72388, Sakaka, Saudi Arabia*

[2]*Engineering Laboratory of Industrial Systems and Renewable Energies, National Higher School of Engineers of Tunis, Tunis University, 1008, Tunis, Tunisia*

[3]*Laboratory of Analysis, Conception and Control of Systems (LACCS), National School of Engineers of Tunis, Tunis El Manar University, 1002, Tunis, Tunisia*

ARTICLE INFO

Article history:
Received: 9 April, 2020
Accepted: 1 June, 2020
Online: 6 July, 2020

Keywords:
TS fuzzy systems
Sliding mode observer
Fault estimation
Fault-tolerant control

ABSTRACT

In this paper, the problems of actuator and sensor fault estimation (FE) and fault-tolerant control (FTC) for uncertain nonlinear systems represented by Takagi-Sugeno (T-S) fuzzy models are investigated. First, a robust fuzzy adaptive sliding mode observer (SMO) is designed to simultaneously estimate system states and both actuator and sensor faults. Then, using the obtained on-line FE information, a static output-feedback fault-tolerant control (SOFFTC) is developed to compensate the fault effects and stabilize the closed-loop system. Moreover, sufficient conditions for the existence of the proposed observer and controller are given in terms of linear matrix inequalities (LMIs). The robustness against uncertainties is treated using the H_∞ optimization technique to attenuate its effect on the estimation error. Finally, the simulation results of nonlinear inverted pendulum with cart system validate the efficiency of the proposed method.

1 Introduction

Modern industrial systems are affected usually by various event of faults such as, loss of actuator effectiveness, failures or offsets of actuators/sensors, deviations of output measurement, etc. Indeed, the presence of fault causes an unacceptable performances of design controllers, thus deteriorating the overall system execution, and so leading to wrong dangerous situations.

Thus, it is important to encourage the development of research on fault tolerant control (FTC), which is divided on two types. The first one, the so-called passive FTC, is focused on to conceive a robust controller against disturbances and uncertainties. A key limitation is that the system stability cant be guaranteed in the presence of faults.

Nevertheless, based on online fault estimation (FE), such as the size and the shape, active FTC can develop robust controller such that the fault effects are eliminated and the system stability is achieved. In the literature, several research results on the FTC techniques are documented, see for example [1–9], and the references therein.

In industrial processes, most of systems are described by nonlinear mathematical models. Takagi-Sugeno (TS) fuzzy systems [10] provide a powerful tool to approximate nonlinear characteristics. T-S fuzzy systems are nonlinear models represented by a set of local linear models. By fuzzy blending of linear representations with appropriate membership functions, the overall fuzzy model of the system is achieved, which greatly simplifies the analysis and control for complex nonlinear systems. Therefore, excellent results in FE and FTC problems of T-S fuzzy systems are developed in [11–17]. In [18], a FTC is designed for TS fuzzy systems subject to actuator faults. However, this result must verify the rank condition, which is really hard to fulfill for many practical systems. In [19], the problem of FE and FTC for a T-S fuzzy systems with uncertainties and actuator faults is investigated without the requirement of rank condition. It deals only with constant faults, however, the faults are time-varying in many real systems. In [20], a sliding mode observer (SMO) is designed to estimate sensor fault for nonlinear stochastic systems for FTC. However, the actuator fault is not con-

*Corresponding Author: Slim Dhahri, Department of Computer Engineering and Networks, College of Computer and Information Sciences, Jouf University, Sakaka, Saudi Arabia, dhahri_slim@yahoo.fr

sidered. In [21], a FTC is developed for T-S fuzzy systems affected by actuator faults. Since in many practical systems, actuator and sensor faults may occur at the same time and uncertainties may exist. It is desirable to consider actuator and sensor faults under one unified framework. For example, in [22], a SMO-based FE and FTC is designed for a class of nonlinear systems subject to actuator and sensor faults. A major disadvantage of this approach is the appearance of chattering mode. In [23], FE and FTC problems are studied to simultaneously estimate sensor and actuator faults using two observers and filter. However, this method is more expensive since it requires a high additional computation cost. In [24], a robust adaptive observer is developed to simultaneously estimate state and both sensor and actuator faults for nonlinear systems despite the presence of disturbances. A FTC law is applied to stabilize the closed-loop system and compensate the fault effects. However, sufficient conditions of observer and controller gains are formulated in an unified optimization problem and computed by solving a set of LMIs only in single step. Unfortunately, these results need the knowledge of the upper bounds of faults. If the information of fault is unknown, the SMO cannot be obtained.

The aim of this work is to address fault estimation and fault tolerant control problems for T-S fuzzy systems subject to simultaneously actuator faults, sensor faults and uncertainties. First, a novel robust adaptive SMO is proposed to estimate the states and both actuator and sensor faults using equivalent output error injection approach. Then, based on online fault information a static output-feedback fault-tolerant control (SOFFTC) is designed to compensate the fault effects and stabilize the closed-loop system. All the design conditions are formulated in an optimization problem under LMIs constraints. Finally, the simulation result of an inverted pendulum with cart system is given to prove the effectiveness of the proposed method.

The main contributions of the present work are the following:

1. A novel fuzzy adaptive SMO is designed for the estimation of states and faults in a T-S fuzzy systems affected by simultaneously actuator faults, sensor faults and uncertainties. Robustness against uncertainties is analyzed using the H_∞ technique to reduce its effect.

2. Most existing SMO design methods such as those reported in [22–24] assume that the value of the upper bounds of actuator faults ρ_a and sensor faults ρ_s is known. If the information of fault is unknown or exceeds the admissible value, these methods cannot be feasible. To overcome this problem, a new adaptive law is constructed to estimate the upper bounds online.

3. The problem of both actuator and sensor FE under one unified framework for T-S fuzzy systems is investigated. Whereas, many researchers have considered only sensor faults [25–27] or actuator faults [28, 29].

4. Basedon the FE, a SOFFTC is designed to effectively accommodate the influence of fault and ensure the stability of the resulting closed-loop system. The proposed method is easily be implemented in practice and is much simpler than dynamic output feedback fault tolerant controller.

5. Sufficient conditions of the observer and controller are formulated in an optimization problem under LMIs constraints which can be designed separately.

The rest of this paper is organized as follows: Section 2 presents the problem formulation and preliminaries. The design of the observer and the analysis of the stability of the error dynamics are given in Section 3. FE is studied in Section 4. Section 5 gives the SOFFTC scheme. Finally, simulation example in Section 6 validates the efficiency of the proposed algorithm.

2 Problem Formulation and Preliminaries

Consider a TS fuzzy model with actuator faults, sensor faults and uncertainties. The ith rule of the T-S fuzzy model is of the following form:

Plant Rule i: If $\xi_1(t)$ is $\mu_{1,i}$ and ... $\xi_g(t)$ is $\mu_{g,i}$, Then

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + M_i f_a(t) + E_i d(x,u,t) \\ y(t) = C_i x(t) + N f_s(t) \\ y_c(t) = C_{ci} x(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ represents the state vector; $u(t) \in R^m$ is the input; $y(t) \in R^p$ is the output; $y_c(t) \in R^{p1}$ is the controlled output; A_i, B_i, M_i, E_i, C_i, N and C_{ci} are real known constant matrices with appropriate dimensions; $f_a(t) : R^+ \to R^q$ and $f_s(t) : R^+ \to R^h$ represent additive actuator fault and sensor fault vector, respectively; $d(x,u,t) \in R^l$ models the uncertainties, which is assumed to belong to $\mathcal{L}_2\,[0,\infty)$; the pairs (A_i, C_i) are observable, and the pairs (A_i, B_i) are controllable; $\xi_j (j = 1, ..., g)$ are the premise variables, and $\mu_{j,i} (j = 1, ..., g; i = 1, ..., k)$ are fuzzy sets; g and k are the number of premise variables and IF-THEN rules, respectively. The fuzzy model is given by:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{k} \mu_i\left(\xi(t)\right)\left(A_i x(t) + B_i u(t) + M_i f_a(t) + E_i d(x,u,t)\right) \\ y(t) = \sum\limits_{i=1}^{k} \mu_i\left(\xi(t)\right)\left(C_i x(t) + N f_s(t)\right) \\ y_c(t) = \sum\limits_{i=1}^{k} \mu_i\left(\xi(t)\right)\left(C_{ci} x(t)\right) \end{cases} \tag{2}$$

where $\xi(t) = \left[\xi_1(t), ..., \xi_g(t)\right]$, $\mu_i\left(\xi(t)\right) = \frac{w_i(\xi(t))}{\sum_{i=1}^{k} w_i(\xi(t))}$, $w_i\left(\xi(t)\right) = \prod\limits_{j=1}^{g} \theta_{ij}\left(\mu_j(t)\right)$ and here $\theta_{ij}(.)$ stands for the order of the membership function of θ_{ij}. It is assumed that

$$w_i\left(\xi(t)\right) \geq 0, \quad i = 1, ..., k, \quad \sum_{i=1}^{k} w_i\left(\xi(t)\right) > 0 \tag{3}$$

for any $\xi(t)$. Thus, for any $\xi(t)$, $\sum\limits_{i=1}^{k} \mu_i\left(\xi(t)\right)$ satisfies

$$\mu_i\left(\xi(t)\right) \geq 0, , \quad i = 1, ..., k, \quad \sum_{i=1}^{k} \mu_i\left(\xi(t)\right) = 1 \tag{4}$$

For simplicity, we will use μ_i to represent $\mu_i\left(\xi(t)\right)$.

Assumption 1. $f_a(t)$ and $f_s(t)$ are unknown but norm bounded

$$\|f_a(t)\| \leq \rho_a, \quad \|f_s(t)\| \leq \rho_s \tag{5}$$

where ρ_a and ρ_s are unknown positive scalars.

Assumption 2 [30]. The actuator fault distribution matrices M_i in (2) satisfy:

$$rank\left(CM_i\right) = rank\left(M_i\right), \ i = 1, ..., k \tag{6}$$

Assumption 3 [30].

$$rank\begin{bmatrix} sI_n - A_i & M_i \\ C_i & 0 \end{bmatrix} = n + rank(M_i), \ i = 1, ..., k \tag{7}$$

Lemma 1 [31]. For matrices A and B and any scalar $\varepsilon > 0$, we have

$$AB + (AB)^T \leq \varepsilon^{-1}AA^T + \varepsilon B^T B \tag{8}$$

Lemma 2 [32]. If

$$S_{ii} < 0, \quad 1 \leq i \leq k \tag{9}$$

$$\frac{2}{r-1}S_{ii} + S_{ij} + S_{ji} < 0, \quad 1 \leq i \neq j \leq k \tag{10}$$

then, we have

$$\sum_{i=1}^{k}\sum_{j=1}^{k}\mu_i\mu_j S_{ij} < 0 \tag{11}$$

Lemma 3 [33]. Under Assumption 2, there exists coordinate transformations

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = T_i x(t), \ \upsilon(t) = \begin{bmatrix} \upsilon_1(t) \\ \upsilon_2(t) \end{bmatrix} = S_i y(t)$$

such that

$$\begin{aligned} T_i A_i T_i^{-1} &= \begin{bmatrix} A_{11,i} & A_{12,i} \\ A_{21,i} & A_{22,i} \end{bmatrix}, T_i B_i = \begin{bmatrix} B_{1,i} \\ B_{2,i} \end{bmatrix}, T_i M_i = \begin{bmatrix} M_{1,i} \\ 0 \end{bmatrix} \\ T_i E_i &= \begin{bmatrix} E_{1,i} \\ E_{2,i} \end{bmatrix}, S_i N = \begin{bmatrix} 0 \\ N_2 \end{bmatrix}, S_i C_i T_i^{-1} = \begin{bmatrix} C_{11,i} & 0 \\ 0 & C_{22,i} \end{bmatrix} \end{aligned}$$

where $A_{11,i} \in R^{q\times q}$, $A_{22,i} \in R^{(n-q)\times(n-q)}$, $B_{1,i} \in R^{q\times m}$, $M_{1,i} \in R^{q\times q}$, $E_{1,i} \in R^{q\times l}$, $N_2 \in R^{(p-q)\times h}$, $C_{11,i} \in R^{q\times q}$ and $C_{22,i} \in R^{(p-q)\times(p-q)}$ is invertible, $i = 1, ..., k$.

Through coordinate transformations, the system (2) is converted into the following two subsystems:

$$\begin{cases} \dot{z}_1(t) = \sum\limits_{i=1}^{k}\mu_i\left(A_{11,i}z_1(t) + A_{12,i}z_2(t) + B_{1,i}u(t) + M_{1,i}f_a(t)\right. \\ \qquad\qquad \left.+E_{1,i}d(x,u,t)\right) \\ \upsilon_1(t) = \sum\limits_{i=1}^{k}\mu_i\left(C_{11,i}z_1(t)\right) \end{cases} \tag{12}$$

$$\begin{cases} \dot{z}_2(t) = \sum\limits_{i=1}^{k}\mu_i\left(A_{21,i}z_1(t) + A_{22,i}z_2(t) + B_{2,i}u(t)\right. \\ \qquad\qquad \left.+E_{2,i}d(x,u,t)\right) \\ \upsilon_2(t) = \sum\limits_{i=1}^{k}\mu_i\left(C_{22,i}z_2(t) + N_2 f_s(t)\right) \end{cases} \tag{13}$$

In addition, partition the matrix S_i as:

$$S_i = \begin{bmatrix} S_{11,i} \\ S_{22,i} \end{bmatrix} \tag{14}$$

where $S_{11,i} \in R^{(p-q)\times p}$ and $S_{22,i} \in R^{q\times p}$. The variable $z_1(t)$ can be obtained by:

$$z_1(t) = \sum_{i=1}^{k}\mu_i\left(C_{11,i}^{-1}S_{11,i}y(t)\right) \tag{15}$$

We define a new state $z_3(t) = \int_0^t \upsilon_2(\tau)d\tau$ where $\dot{z}_3(t) = \sum\limits_{i=1}^{k}\mu_i\left(C_{22,i}z_2(t) + N_2 f_s(t)\right)$. Then the augmented system with the new state $z_3(t)$ is given as:

$$\begin{cases} \dot{z}_0(t) = \sum\limits_{i=1}^{k}\mu_i\left(A_{0,i}z_0(t) + A_{3,i}z_2(t) + B_{0,i}u(t) + M_{0,i}f_s(t)\right. \\ \qquad\qquad \left.+E_{0,i}d(x,u,t)\right) \\ \upsilon_3(t) = \sum\limits_{i=1}^{k}\mu_i\left(C_{0,i}z_0(t)\right) \end{cases} \tag{16}$$

where $\dot{z}_0(t) = \begin{bmatrix} z_2(t) \\ z_3(t) \end{bmatrix} \in R^{n+p-2q}$, $\upsilon_3(t) \in R^{p-q}$, $A_{0,i} = \begin{bmatrix} A_{22,i} & 0 \\ C_{22,i} & 0 \end{bmatrix} \in R^{(n+p-2q)\times(n+p-2q)}$, $A_{3,i} = \begin{bmatrix} A_{21,i} \\ 0 \end{bmatrix} \in R^{(n+p-2q)\times q}$, $B_{0,i} = \begin{bmatrix} B_{2,i} \\ 0 \end{bmatrix} \in R^{(n+p-2q)\times m}$, $E_{0,i} = \begin{bmatrix} E_{2,i} \\ 0 \end{bmatrix} \in R^{(n+p-2q)\times l}$, $M_{0,i} = \begin{bmatrix} 0 \\ N_2 \end{bmatrix} \in R^{(n+p-2q)\times h}$ and $C_{0,i} = \begin{bmatrix} 0 & I_{p-q} \end{bmatrix} \in R^{(p-q)\times(n+p-2q)}$.

Lemma 4 [33]. The pair $(A_{0,i}, C_{0,i})$ is observable, if the pair $(A_{22,i}, C_{22,i})$ is detectable, $i = 1, ..., k$. Then, there exists matrices L_i, having the special structure $L_i = \begin{bmatrix} L_{1,i} & 0 \end{bmatrix}$, such that $A_{22,i} + L_i C_{22,i}$ is stable, $i = 1, ..., k$.

Let the transformation of coordinates $h(t) = \begin{bmatrix} h_1^T(t) & h_2^T(t) \end{bmatrix}^T = T_{L,i}z_0(t)$ with

$$T_{L,i} = \begin{bmatrix} I_{n-q} & L_i \\ 0 & I_{p-q} \end{bmatrix}, \quad i = 1, ..., k \tag{17}$$

where $h_1(t) \in R^{n-q}$ and $h_2(t) \in R^{p-q}$. Therefore, the system (16) is converted into the following system:

$$\begin{cases} \dot{h}(t) = \sum\limits_{i=1}^{k}\mu_i\left(A_{h,i}h(t) + T_{L,i}A_{3,i}z_2(t) + B_{h,i}u(t)\right. \\ \qquad\qquad \left.+E_{h,i}d(x,u,t) + M_{h,i}f_s(t)\right) \\ \upsilon_3(t) = \sum\limits_{i=1}^{k}\mu_i\left(C_{h,i}h(t)\right) \end{cases} \tag{18}$$

where

$$\begin{aligned} A_{h,i} &= \begin{bmatrix} A_{22,i} + L_i C_{22,i} & -(A_{22,i} + L_i C_{22,i})L_i \\ C_{22,i} & -C_{22,i}L_i \end{bmatrix}, B_{h,i} = \begin{bmatrix} B_{1,i} \\ 0 \end{bmatrix} \\ E_{h,i} &= \begin{bmatrix} E_{1,i} \\ 0 \end{bmatrix}, M_{h,i} = \begin{bmatrix} 0 \\ N_2 \end{bmatrix}, C_{h,i} = \begin{bmatrix} 0 & I_{p-q} \end{bmatrix} \end{aligned}$$

Therefore, T-S fuzzy subsystems (12) and (13) can be rewritten respectively as:

$$\begin{cases} \dot{z}_1(t) = \sum\limits_{i=1}^{k}\mu_i\left(A_{11,i}z_1(t) + A_{12,i}z_2(t) + B_{1,i}u(t) + M_{1,i}f_a(t)\right. \\ \qquad\qquad \left.+E_{1,i}d(x,u,t)\right) \\ \upsilon_1(t) = \sum\limits_{i=1}^{k}\mu_i\left(C_{11,i}z_1(t)\right) \end{cases} \tag{19}$$

$$\begin{cases} \dot{h}_1(t) = \sum_{i=1}^{k} \mu_i \left((A_{22,i} + L_i C_{22,i}) h_1(t) - (A_{22,i} + L_i C_{22,i}) L_i h_2(t) \right. \\ \qquad \left. + A_{12,i} z_1(t) + B_{1,i} u(t) + E_{1,i} d(x,u,t) \right) \\ \dot{h}_2(t) = \sum_{i=1}^{k} \mu_i \left(C_{22,i} h_1(t) - C_{22,i} L_i h_2(t) + N_2 f_s(t) \right) \\ \upsilon_3(t) = h_2(t) \end{cases} \tag{20}$$

3 Adaptive Sliding Mode Observers Design

For system (19), we construct the following adaptive SMO:

$$\begin{cases} \dot{\hat{z}}_1(t) = \sum_{i=1}^{k} \mu_i \Big(A_{11,i} \hat{z}_1(t) + A_{12,i} \hat{h}_1(t) - A_{12,i} L_i \upsilon_3(t) \\ \quad + B_{1,i} u(t) + M_{1,i} \nu_{1,i}(t) + (A_{11,i} - A^s_{11,i}) C^{-1}_{11,i} (\upsilon_1(t) - \hat{\upsilon}_1(t)) \Big) \\ \hat{\upsilon}_1(t) = \sum_{i=1}^{k} \mu_i \left(C_{11,i} \hat{z}_1(t) \right) \end{cases} \tag{21}$$

where $\hat{z}_1(t)$, $\hat{h}_1(t)$ and $\hat{\upsilon}_1(t)$ denote, respectively, the estimated $z_1(t)$, $h_1(t)$ and $\upsilon_1(t)$. $A^s_{11,i} \in R^{q\times q}$ is a stable matrix and $\nu_{1,i}(t)$ is defined by:

$$\nu_{1,i}(t) = \begin{cases} (\hat{\rho}_a + l_{a,i}) \frac{M^T_{1,i} P_1 \left(C^{-1}_{11,i} S_{11,i} \upsilon_1(t) - \hat{z}_1(t) \right)}{\left\| M^T_{1,i} P_1 \left(C^{-1}_{11,i} S_{11,i} \upsilon_1(t) - \hat{z}_1(t) \right) \right\|} & \text{if } C^{-1}_{11,i} S_{11,i} \upsilon_1 - \hat{z}_1 \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $P_1 \in R^{q\times q} > 0$ is the Lyapunov matrix for $A^s_{11,i}$, $\hat{\rho}_a$ is adaptive parameter to estimate the unknown parameter ρ_a, and the scalar $\hat{\rho}_a$ is introduced using an update law

$$\dot{\hat{\rho}}_a = \sigma_1 \left\| M^T_{1,i} P_1 \left(C^{-1}_{11,i} S_{11,i} \upsilon_1(t) - \hat{z}_1(t) \right) \right\| \tag{22}$$

with constant $\sigma_1 > 0$.
For system (20), we design the following adaptive SMO:

$$\begin{cases} \dot{\hat{h}}_1(t) = \sum_{i=1}^{k} \mu_i \Big((A_{22,i} + L_i C_{22,i}) \hat{h}_1(t) - (A_{22,i} + L_i C_{22,i}) \\ \qquad \times L_i \upsilon_3(t) + B_{1,i} u(t) + A_{21,i} C^{-1}_{11,i} \upsilon_1(t) \Big) \\ \dot{\hat{h}}_2(t) = \sum_{i=1}^{k} \mu_i \Big(C_{22,i} \hat{h}_1(t) - C_{22,i} L_i \hat{h}_2(t) \\ \qquad - (C_{22,i} L_i + K_i) (\upsilon_3(t) - \hat{\upsilon}_3(t)) + N_2 \nu_{2,i}(t) \Big) \\ \hat{\upsilon}_3(t) = \hat{h}_2(t) \end{cases} \tag{23}$$

where $\hat{h}_1(t)$ and $\hat{\upsilon}_3(t)$ denote, respectively, the estimated of $h_1(t)$ and $\upsilon_3(t)$, $K_i \in R^{(p-q)\times(p-q)}$ is the observer gains, and $\nu_{2,i}(t)$ is defined by:

$$\nu_{2,i}(t) = \begin{cases} (\hat{\rho}_s + l_{s,i}) \frac{N^T_2 P_{02} (\upsilon_3(t) - \hat{\upsilon}_3(t))}{\| N^T_2 P_{02} (\upsilon_3(t) - \hat{\upsilon}_3(t)) \|} & \text{if } \upsilon_3(t) - \hat{\upsilon}_3(t) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $P_{02} \in R^{(p-q)\times(p-q)} > 0$, $\hat{\rho}_s$ is adaptive parameter to estimate the unknown parameter ρ_s, and the scalar $\hat{\rho}_s$ is introduced using an update law

$$\dot{\hat{\rho}}_s = \sigma_2 \left\| N^T_2 P_{02} (\upsilon_3(t) - \hat{\upsilon}_3(t)) \right\| \tag{24}$$

with constant $\sigma_2 > 0$.

Let us define $e_1(t) = z_1(t) - \hat{z}_1(t)$, $e_2(t) = h_1(t) - \hat{h}_1(t)$ and $e_3(t) = h_2(t) - \hat{h}_2(t)$, then the error dynamic system as follows:

$$\begin{cases} \dot{e}_1(t) = \sum_{i=1}^{k} \mu_i \left(A_{11,i} e_1(t) + A_{12,i} e_2(t) + E_{1,i} d(x,u,t) \right. \\ \qquad \left. + M_{1,i} (f_a(t) - \nu_{1,i}(t)) \right) \\ \dot{e}_2(t) = \sum_{i=1}^{k} \mu_i \left((A_{22,i} + L_i C_{22,i}) e_2(t) + E_{2,i} d(x,u,t) \right) \\ \dot{e}_3(t) = \sum_{i=1}^{k} \mu_i \left(C_{22,i} e_2(t) + K_i e_3(t) + N_2 (f_s(t) - \nu_{2,i}(t)) \right) \end{cases} \tag{25}$$

Define $r(t)$ as

$$r(t) = He(t) = H \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} \tag{26}$$

where

$$H := \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_2 & 0 \\ 0 & 0 & H_3 \end{bmatrix} \tag{27}$$

with $H_1 \in R^{q\times q}$, $H_2 \in R^{(n-q)\times(n-q)}$ and $H_3 \in R^{(p-q)\times(p-q)}$. The adaptive SMO design method under H_∞ performance to be addressed in this work is

(i) The observer error dynamics system (25) with $d(x,u,t) = 0$ is asymptotically stable, namely, there is no uncertainty;

(ii) For a given $\gamma_1 > 0$. The following H_∞ performance is satisfied:

$$\int_0^T r^T(t) r(t) dt < \gamma_1 \int_0^T d^T(x,u,t) d(x,u,t) dt \tag{28}$$

for all $T > 0$ and $d(x,u,t) \in \mathcal{L}_2 \left[\begin{array}{cc} 0 & \infty \end{array} \right)$ under zero initial conditions.

3.1 Stability analysis

Theorem 1. Consider T-S fuzzy system (2) under Assumptions 13. The observer error dynamics system (25) is asymptotically stable and satisfy (28) with attenuation level $\gamma_1 > 0$, if there exist matrices $P_1 > 0$, $P_{01} > 0$, $P_{02} > 0$, X_i, Y_i, $i = 1, ..., k$, such that:

Minimize γ_1 subject to

$$\begin{bmatrix} \Gamma_{1,i} & P_1 A_{12,i} & 0 & P_1 E_{1,i} \\ * & \Gamma_{2,i} & C^T_{22,i} P_{02} & P_{01} E_{2,i} \\ * & * & \Gamma_{3,i} & 0 \\ * & * & * & -\gamma_1 I \end{bmatrix} < 0 \tag{29}$$

where

$$\begin{aligned} \Gamma_{1,i} &= (A^s_{11,i})^T P_1 + P_1 A^s_{11,i} + H^T_1 H_1 \\ \Gamma_{2,i} &= A_{22,i} P_{01} + P_{01} A^T_{22,i} + X_i C_{22,i} + C^T_{22,i} X^T_i + H^T_2 H_2 \\ \Gamma_{3,i} &= Y_i + Y^T_i + H^T_3 H_3 \end{aligned}$$

If the optimization problem is solved, then we can obtain the following observer gains

$$\begin{aligned} L_i &= P^{-1}_{01} X_i \\ K_i &= P^{-1}_{02} Y_i \end{aligned}$$

Proof. Let the following Lyapunov functional candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{30}$$

where $V_1(t) = e_1^T(t)P_1e_1(t) + \frac{1}{\sigma_1}\tilde{\rho}_a^2$, $V_2(t) = e_2^T(t)P_{01}e_2(t)$, $V_3(t) = e_3^T(t)P_{02}e_3(t) + \frac{1}{\sigma_2}\tilde{\rho}_s^2$, $\tilde{\rho}_a = \rho_a - \hat{\rho}_a$ and $\tilde{\rho}_s = \rho_s - \hat{\rho}_s$. The derivative of $V_1(t)$ satisfy:

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^{k}\mu_i\Big(e_1^T(t)\left((A_{11,i}^s)^TP_1 + P_1A_{11,i}^s\right)e_1(t)\\ &\quad +2e_1^T(t)P_1A_{12,i}e_2(t) + 2e_1^T(t)P_1E_{1,i}d(x,u,t)\\ &\quad +2e_1^T(t)P_1M_{1,i}(f_a(t) - \nu_{1,i}(t))\Big) + \frac{2}{\sigma_1}\tilde{\rho}_a(-\dot{\hat{\rho}}_a)\end{aligned} \tag{31}$$

Using the definition of $\nu_{1,i}(t)$ and the bound of $f_a(t)$, we have

$$\begin{aligned}& e_1^T(t)P_1M_{1,i}(f_a(t) - \nu_{1,i}(t)) + \frac{1}{\sigma_1}\tilde{\rho}_a(-\dot{\hat{\rho}}_a)\\ =\;& e_1^T(t)P_1M_{1,i}f_a(t) - (\hat{\rho}_a + l_{a,i})\,e_1^T(t)P_1M_{1,i}\frac{M_{1,i}^TP_1e_1(t)}{\left\|M_{1,i}^TP_1e_1(t)\right\|}\\ & +\frac{1}{\sigma_1}(\rho_a - \hat{\rho}_a)\left(-\sigma_1\left\|M_{1,i}^TP_1e_1\right\|\right)\\ =\;& e_1^T(t)P_1M_{1,i}f_a(t) - (\rho_a + l_{a,i})\left\|M_{1,i}^TP_1e_1(t)\right\|\\ \le\;& \left\|M_{1,i}^TP_1e_1(t)\right\|\rho_a - (\rho_a + l_{a,i})\left\|M_{1,i}^TP_1e_1(t)\right\|\\ =\;& -l_{a,i}\left\|M_{1,i}^TP_1e_1(t)\right\| < 0\end{aligned} \tag{32}$$

Therefore

$$\begin{aligned}\dot{V}_1(t) &\le \sum_{i=1}^{k}\mu_i\Big(e_1^T(t)\left((A_{11,i}^s)^TP_1 + P_1A_{11,i}^s\right)e_1(t)\\ &\quad +2e_1^T(t)P_1A_{12,i}e_2(t) + 2e_1^T(t)P_1E_{1,i}d(x,u,t)\end{aligned} \tag{33}$$

Similarly, the derivatives of $V_2(t)$ and $V_3(t)$ can be obtained as:

$$\begin{aligned}\dot{V}_2(t) &= \sum_{i=1}^{k}\mu_i\Big(e_2^T(t)\left((A_{22,i} + L_iC_{22,i})^TP_{01}\right.\\ &\quad \left.+P_{01}(A_{22,i} + L_iC_{22,i})\right)e_2(t)\\ &\quad +2e_2^T(t)P_{01}E_{2,i}d(x,u,t)\Big)\end{aligned} \tag{34}$$

$$\begin{aligned}\dot{V}_3(t) &= \sum_{i=1}^{k}\mu_i\Big(e_3^T(t)\left(K_i^TP_{02} + P_{02}K_i\right)e_3(t)\\ &\quad +2e_3^T(t)P_{02}C_{22,i}e_2(t)\\ &\quad +2e_3^T(t)P_{02}N_2(f_s(t) - \nu_{2,i}(t))\Big)\end{aligned} \tag{35}$$

Similarly, we obtain

$$2e_3^T(t)P_{02}N_2(f_s(t) - \nu_{2,i}(t)) \le -l_{s,i}\left\|N_2^TP_{02}e_3(t)\right\| < 0 \tag{36}$$

From (30), (33)(36), the time derivative of $V(t)$ is

$$\begin{aligned}\dot{V}(t) &\le \sum_{i=1}^{k}\mu_i(\xi(t))\Big(e_1^T(t)\left((A_{11,i}^s)^TP_1 + P_1A_{11,i}^s\right)e_1(t)\\ &\quad +2e_1^T(t)P_1A_{12,i}e_2(t) + 2e_1^T(t)P_1E_{1,i}d(x,u,t)\\ &\quad +e_2^T(t)\left((A_{22,i} + L_iC_{22,i})^TP_{01} + P_{01}(A_{22,i} + L_iC_{22,i})\right)e_2(t)\\ &\quad +2e_2^T(t)P_{01}E_{2,i}d(x,u,t) + e_3^T(t)\left(K_i^TP_{02} + P_{02}K_i\right)e_3(t)\\ &\quad +2\,e_3^T(t)P_{02}C_{22,i}e_2(t)\Big)\end{aligned} \tag{37}$$

When $d(x,u,t) = 0$, we have

$$\dot{V}(t) \le \sum_{i=1}^{k}\mu_i\left(\begin{bmatrix} e_1(t)\\ e_2(t)\\ e_3(t)\end{bmatrix}^T \Lambda_i \begin{bmatrix} e_1(t)\\ e_2(t)\\ e_3(t)\end{bmatrix}\right) \tag{38}$$

where

$$\Lambda_i = \begin{bmatrix} Q_{1,i} & P_1A_{12,i} & 0\\ A_{12,i}^TP_1 & Q_{2,i} & C_{22,i}^TP_{02}\\ 0 & P_{02}C_{22,i} & Q_{3,i}\end{bmatrix} \tag{39}$$

with

$$\begin{aligned}Q_{1,i} &= (A_{11,i}^s)^TP_1 + P_1A_{11,i}^s\\ Q_{2,i} &= (A_{22,i} + L_iC_{22,i})^TP_{01} + P_{01}(A_{22,i} + L_iC_{22,i})\\ Q_{3,i} &= K_i^TP_{02} + P_{02}K_i\end{aligned}$$

If $\Lambda_i < 0$, then $\dot{V}(t) < 0$, which implies that $e \to 0$ as $t \to \infty$. Therefore, the error dynamics system is asymptotically stable.

When $d(x,u,t) \ne 0$, we define

$$J_1(t) = \dot{V}(t) + r^T(t)r(t) - \gamma_1 d^T(x,u,t)d(x,u,t) \tag{40}$$

Substituting (37) and (26) into (28) yields

$$\begin{aligned}J_1(t) &= \dot{V}(t) + r^T(t)r(t) - \gamma_1 d^T(x,u,t)d(x,u,t)\\ &= \sum_{i=1}^{k}\mu_i\Big(e^T\left(\Lambda_i + H^TH\right)e + +2e_1^T(t)P_1E_{1,i}d(x,u,t)\\ &\quad +2e_2^T(t)P_{01}E_{2,i}d(x,u,t) - \gamma_1 d^T(x,u,t)d(x,u,t)\Big)\\ &= \sum_{i=1}^{k}\mu_i\Bigg(\begin{bmatrix} e_1(t)\\ e_2(t)\\ e_3(t)\end{bmatrix}^T \Lambda_i \begin{bmatrix} e_1(t)\\ e_2(t)\\ e_3(t)\end{bmatrix}\\ &\quad +2\begin{bmatrix} e_1(t)\\ e_2(t)\\ e_3(t)\end{bmatrix}^T \begin{bmatrix} P_1 & 0 & 0\\ 0 & P_{01} & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} E_{1,i}\\ E_{2,i}\\ 0\end{bmatrix} d(x,u,t)\\ &\quad -\gamma_1 d^T(x,u,t)d(x,u,t)\Bigg)\\ &= \sum_{i=1}^{k}\mu_i\left(\begin{bmatrix} e_1(t)\\ e_2(t)\\ e_2(t)\\ d(x,u,t)\end{bmatrix}^T \psi_i \begin{bmatrix} e_1(t)\\ e_2(t)\\ e_2(t)\\ d(x,u,t)\end{bmatrix}\right)\end{aligned} \tag{41}$$

with

$$\psi_i = \begin{bmatrix} Q_{1,i} + H_1^TH_1 & P_1A_{12,i} & 0 & P_1E_{1,i}\\ * & Q_{2,i} + H_2^TH_2 & C_{22,i}^TP_{02} & P_{01}E_{2,i}\\ * & * & Q_{3,i} + H_3^TH_3 & 0\\ * & * & * & -\gamma_1 I\end{bmatrix} \tag{42}$$

The previous inequalities are nonlinear because of $P_{01}L_i$ and $P_{02}K_i$. This problem can be solved using the variable change $X_i = P_{01}L_i$ and $Y_i = P_{02}K_i$. Applying the Schur complement, we can obtain the LMI form (29). So if $J_1(t) < 0$, the error dynamics system (25) is stable satisfying the H_∞ performance (28). □.

3.2 Sliding motion analysis

For system (25), Let

$$\mathcal{S} = \{(e_1(t), e_2(t), e_3(t)) \,|e_1(t) = 0, e_3(t) = 0\} \quad (43)$$

Theorem 2. Consider system (2) satisfying Assumptions 13. The system error dynamics (25) can be driven to the sliding surface $\mathcal{S}$ in finite time and remain on it if the LMI (29) is solvable and the gains $l_{a,i}$ and $l_{s,i}$ satisfy:

$$l_{a,i} \geq \left\|M_{1,i}^{-T}\right\| \left(\left\|A_{12,i}\right\| \|e_2(t)\| + \left\|E_{1,i}\right\| \|d(x,u,t)\|\right)\right) + \eta_{a,i} \quad (44)$$

$$l_{s,i} \geq \left\|N_2^{-T}\right\| \left\|C_{22,i}\right\| \|e_2(t)\| + \eta_{s,i} \quad (45)$$

where $\eta_{a,i}$ and $\eta_{s,i}$ are two positive scalars, $i = 1, ..., k$.

Proof:
Consider $V_1(t) = e_1^T(t)P_1e_1(t) + \frac{1}{\sigma_1}\tilde{\rho}_a^2$ and $V_3(t) = e_3^T(t)P_{02}e_3(t) + \frac{1}{\sigma_2}\tilde{\rho}_s^2$. The differentia of $V_1(t)$ can be obtained as:

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^{k}\mu_i \Big(e_1^T(t)\left((A_{11,i}^s)^T P_1 + P_1 A_{11,i}^s\right) e_1(t) \\ &\quad +2e_1^T(t)P_1A_{12,i}e_2(t) + 2e_1^T(t)P_1E_{1,i}d(x,u,t) \\ &\quad +2e_1^T(t)P_1M_{1,i}(f_a(t) - \nu_{1,i}(t))\Big) + \frac{2}{\sigma_1}\tilde{\rho}_a(-\dot{\hat{\rho}}_a) \quad (46)\end{aligned}$$

Since $(A_{11,i}^s)^T P_1 + P_1 A_{11,i}^s < 0$, then we can get

$$\begin{aligned}\dot{V}_1(t) &\leq \sum_{i=1}^{k}\mu_i \Big(2e_1^T(t)P_1A_{12,i}e_2(t) + 2e_1^T(t)P_1E_{1,i}d(x,u,t) \\ &\quad -l_{a,i}\left\|M_{1,i}^T P_1 e_1(t)\right\|\Big) \\ &\leq \sum_{i=1}^{k}\mu_i \left[\|P_1e_1(t)\|\left(\left\|A_{12,i}\right\|\|e_2(t)\| + \left\|E_{1,i}\right\|\|d(x,u,t)\|\right)\right. \\ &\quad \left.-l_{a,i}\left\|M_{1,i}^T P_1 e_1(t)\right\|\right] \\ &\leq \sum_{i=1}^{k}\mu_i \left[\left\|M_{1,i}^T P_1 e_1(t)\right\|\left(\left\|M_{1,i}^{-T}\right\|\left(\left\|A_{12,i}\right\|\|e_2(t)\|\right.\right.\right. \\ &\quad \left.\left.\left. + \left\|E_{1,i}\right\|\|d(x,u,t)\|\right)\right) - l_{a,i}\right] \quad (47)\end{aligned}$$

From (46), one obtains

$$\dot{V}_1(t) \leq -2\eta_{a,i}\left\|M_{1,i}^T P_1 e_1(t)\right\| \leq -2\eta_{a,i}\left\|M_{1,i}\right\|\sqrt{\lambda_{\min}(P_1)}V_1^{1/2}(t) \quad (48)$$

If (45) is verified, then

$$\dot{V}_3(t) \leq -2\eta_{s,i}\left\|N_2^T P_{02}e_3(t)\right\| \leq -2\eta_{s,i}\|N_2\|\sqrt{\lambda_{\min}(P_{02})}V_3^{1/2}(t) \quad (49)$$

Then the reachability condition [34] is verified.

4 Fault Estimation

From Theorem 2, an ideal sliding mode take place on $\mathcal{S}$ and $\dot{e}_1(t) = e_1(t) = 0$. Consequently, the error dynamics of $e_1(t)$ becomes:

$$0 = \sum_{i=1}^{k}\mu_i \left(A_{12,i}e_2(t) + E_{1,i}d(x,u,t) + M_{1,i}(f_a(t) - \nu_{1eq,i}(t))\right) \quad (50)$$

where $\nu_{1eq}(t)$ denotes the equivalent term [34] replaced by:

$$\nu_{1eq,i}(t) = (\hat{\rho}_a + l_{a,i})\frac{M_{1,i}^T P_1\left(C_{11,i}^{-1}S_{11,i}\upsilon_1(t) - \hat{z}_1(t)\right)}{\left\|M_{1,i}^T P_1\left(C_{11,i}^{-1}S_{11,i}\upsilon_1(t) - \hat{z}_1(t)\right)\right\| + \delta_a} \quad (51)$$

where $\delta_a > 0$. Since $M_{1,i}$ is invertible, (50) can be rewritten as:

$$\nu_{1eq,i}(t) - f_a(t) = M_{1,i}^{-1}\left(A_{12,i}e_2(t) + E_{1,i}d(x,u,t)\right) \quad (52)$$

Computing the L_2 norm of (52) yields

$$\begin{aligned}&\left\|\nu_{1eq,i}(t) - f_a(t)\right\|_2 \\ =\;& \left\|M_{1,i}^{-1}\left(A_{12,i}e_2(t) + E_{1,i}d(x,u,t)\right)\right\|_2 \\ \leq\;& \left\|M_{1,i}^{-1}A_{12,i}\right\|_2\|e_2(t)\|_2 + \left\|M_{1,i}^{-1}E_{1,i}\right\|_2\|d(x,u,t)\|_2 \\ \leq\;& \left\|M_1^{-1}A_{12}\right\|_{\max}\|e(t)\|_2 + \left\|M_1^{-1}E_1\right\|_{\max}\|d(x,u,t)\|_2 \\ \leq\;& \Big(\sqrt{\gamma_1}\left\|M_1^{-1}A_{12}\right\|_{\max}\sigma_{\max}(H^{-1}) \\ &+\left\|M_1^{-1}E_1\right\|_{\max}\Big)\|d(x,u,t)\|_2 \quad (53)\end{aligned}$$

where $\left\|M_1^{-1}A_{12}\right\|_{\max} = \max\limits_{i=1,\dots,k}\left(\left\|M_{1,i}^{-1}A_{12,i}\right\|_2\right)$ and $\left\|M_1^{-1}E_1\right\|_{\max} = \max\limits_{i=1,\dots,k}\left(\left\|M_{1,i}^{-1}E_{1,i}\right\|_2\right)$. since $\|e(t)\| \leq \sigma_{\max}(H^{-1})\sqrt{\gamma_1}\|d(x,u,t)\|$. It follows that:

$$\sup_{\|d\|\neq 0} = \frac{\left\|\nu_{1eq,i}(t) - f_a(t)\right\|_2}{\|d(x,u,t)\|_2} = \sqrt{\gamma_1}\beta_1 + \beta_2 \quad (54)$$

where $\beta_1 = \left\|M_1^{-1}A_{12}\right\|_{\max}\sigma_{\max}(H^{-1})$ and $\beta_2 = \left\|M_1^{-1}E_1\right\|_{\max}$. Thus for a small $\sqrt{\gamma_1}\beta_1 + \beta_2$, $f_a(t)$ can be estimated as:

$$\hat{f}_a(t) \cong \sum_{i=1}^{k}\mu_i\left((\hat{\rho}_a + l_{a,i})\frac{M_{1,i}^T P_1\left(C_{11,i}^{-1}S_{11,i}\upsilon_1(t) - \hat{z}_1(t)\right)}{\left\|M_{1,i}^T P_1\left(C_{11,i}^{-1}S_{11,i}\upsilon_1(t) - \hat{z}_1(t)\right)\right\| + \delta_a}\right) \quad (55)$$

Similarly, we can get

$$\sup_{\|d\|\neq 0} = \frac{\left\|\nu_{2eq,i}(t) - f_s(t)\right\|_2}{\|d(x,u,t)\|_2} = \sqrt{\gamma_1}\left\|N_2^{-1}C_{22}\right\|_{\max}\sigma_{\max}(H^{-1}) \quad (56)$$

Therefore for small $\sqrt{\gamma_1}\left\|N_2^{-1}C_{22}\right\|_{\max}\sigma_{\max}(H^{-1})$, $f_s(t)$ can be estimated as:

$$\hat{f}_s(t) \cong \sum_{i=1}^{k}\mu_i\left((\hat{\rho}_2 + l_{s,i})\frac{N_{2,i}^T P_{03}e_3(t)}{\left\|N_{2,i}^T P_{03}e_3(t)\right\|}\right) \quad (57)$$

5 Fault Tolerant Controller Design

Define corrected output as:

$$y_c(t) = \sum_{i=1}^{k}\mu_i\left(C_i x(t) + N\left(f_s(t) - \hat{f}_s(t)\right)\right) \quad (58)$$

System (2) becomes:

$$\begin{cases}\dot{x}(t) = \sum\limits_{i=1}^{k}\mu_i\left(A_i x(t) + B_i u(t) + M_i f_a(t) + E_i d(t)\right) \\ y_c(t) = \sum\limits_{i=1}^{k}\mu_i\left(C_i x(t) + N e_{fs}(t)\right)\end{cases} \quad (59)$$

where $e_{f_s(t)} = f_s(t) - \hat{f}_s(t)$.
A SOFFTC law [35] is designed as follows:

$$u(t) = \sum_{i=1}^{k} \mu_i \left(\bar{K}_i y_c(t) - \bar{G}_i \hat{f}_a(t) \right) \tag{60}$$

where K_i and G_i are gains matrices to be determined.
Substituting (60) in (59), we have

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j \Big(A_i x(t) + B_i(\bar{K}_j C_i x(t) \\
&\quad + \bar{K}_j N e_{fs}(t) - \bar{G}_j \hat{f}_a(t)\Big) + M_i f_a(t) + E_i d(x,u,t)) \\
&= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j \Big(A_i x(t) + B_i \bar{K}_j C_i x(t) + B_i \bar{K}_j N e_{fs}(t) \\
&\quad - B_i \bar{G}_j \hat{f}_a(t) + M_i f_a(t) + E_i d(x,u,t) \Big)
\end{aligned} \tag{61}$$

The gain G_i is designed so that $B_i G_i = M_i$ where B_i^+ is the pseudo inverse of B_i [36]. It follows that

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j \Big(\left(A_i + B_i \bar{K}_j C_i \right) x(t) + B_i \bar{K}_j N e_{fs}(t) \\
&\quad + M_i e_{fa}(t) + E_i d(x,u,t) \Big)
\end{aligned} \tag{62}$$

where $e_{fa}(t) = f_a(t) - \hat{f}_a(t)$. Then, we get

$$\begin{cases}
\dot{x}(t) = \sum\limits_{i=1}^{k}\sum\limits_{j=1}^{k} \mu_i\mu_j \left[\left(A_i + B_i \bar{K}_j C_i \right) x(t) + \bar{B}_{ij}\varphi(t) \right] \\
y_c(t) = \sum\limits_{i=1}^{k} \mu_i \left(C_i x(t) + N e_{fs}(t) \right)
\end{cases} \tag{63}$$

where $\bar{B}_{ij} = \begin{bmatrix} B_i \bar{K}_j N & M_i & E_i \end{bmatrix}$ and $\varphi(t) = \begin{bmatrix} e_{fs}^T(t) & e_{fa}^T(t) & d^T(x,u,t) \end{bmatrix}^T$. The control purpose in this paper for the closed-loop fuzzy system (63) is to design a SOFFTC (60) such that

(i) The closed-loop fuzzy system (63) with ($\varphi(t) = 0$) is asymptotically stable .

(ii) For a given scalar $\gamma_2 > 0$, the following H_∞ performance is satisfied:

$$\int_0^L \|y_c(t)\|_2^2 dt < \gamma_2 \int_0^L \|\varphi(t)\|_2^2 dt \tag{64}$$

for all $L > 0$ and $\varphi(t) \in \mathcal{L}_2[0,\infty)$ under zero initial conditions.

Theorem 3. The closed-loop system (63) is asymptotically stable and satisfy the H_∞ performance index (64), if there exist matrix $\bar{P}_x > 0$, matrices R, $\bar{S}_i$ and scalar $\epsilon > 0$, such that:

$$\begin{cases}
\Psi_{ii} < 0, \quad 1 \le i \le k \\
\frac{2}{r-1}\Psi_{ii} + \Psi_{ij} + \Psi_{ji} < 0, \quad 1 \le i \ne j \le k \\
C_i \bar{P}_x = R C_i, \quad 1 \le i \le k
\end{cases} \tag{65}$$

where

$$\Psi_{ij} = \begin{bmatrix}
\Phi_{ij} & 0 & M_i & N & \bar{P}_x C_i^T & B_i \bar{K}_j & 0 \\
* & -2\gamma_2 \bar{P}_x + \gamma_2 I & 0 & 0 & \bar{P}_x N^T & 0 & \varepsilon \bar{P}_x C_i \\
* & * & -\gamma_2 I & 0 & 0 & 0 & 0 \\
* & * & * & -\gamma_2 I & 0 & 0 & 0 \\
* & * & * & * & -\gamma_2 I & 0 & 0 \\
* & * & * & * & * & -\epsilon I & 0 \\
* & * & * & * & * & * & -\epsilon I
\end{bmatrix}$$

with

$$\Phi_{ij} = A_i \bar{P}_x + \bar{P}_x A_i + B_i S_j C_i + B_i^T S_j^T C_i^T$$

The controller gains are obtained by:

$$\bar{K}_i = R^{-1} \bar{S}_i$$

Proof. Choose $V_x(t) = x^T(t) P_x x(t)$, where $P_x > 0$. Its derivative is:

$$\begin{aligned}
\dot{V}_x(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j \Big(x^T(t) \Big((A_i + B_i \bar{K}_j C_i)^T P_x \\
&\quad + P_x (A_i + B_i \bar{K}_j C_i) \Big) x(t) + 2x^T(t) P_x \bar{B}_{ij} \varphi(t) \Big)
\end{aligned} \tag{66}$$

Let

$$J_x(t) = \dot{V}_x(t) + y_c^T(t) y_c(t) - \gamma_c \varphi^T(t)\varphi(t) \tag{67}$$

where

$$\begin{aligned}
y_c^T(t) y_c(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j \Big(\left(C_i x(t) + N e_{fs}(t) \right)^T \\
&\quad \left(C_j x(t) + N e_{fs}(t) \right) \Big) \\
&= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j \Big(x^T(t) C_i^T C_j x(t) + x^T(t) C_i^T N e_{fs}(t) \\
&\quad + e_{fs}^T(t) N^T C_j x(t) + e_{fs}^T(t) N^T N e_{fs}(t) \Big)
\end{aligned} \tag{68}$$

Define $Z = \begin{bmatrix} N & 0 & 0 \end{bmatrix}$, then

$$\begin{aligned}
y_c^T(t) y_c(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j \Big(x^T(t) C_i^T C_j x(t) \\
&\quad + \varphi^T(t) Z^T Z \varphi(t) + 2x^T(t) C_i^T Z \varphi(t) \Big)
\end{aligned} \tag{69}$$

So we obtain

$$\begin{aligned}
J_x(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j \Big(x^T(t) \Big((A_i + B_i \bar{K}_j C_i)^T P_x \\
&\quad + P_x (A_i + B_i \bar{K}_j C_i) \Big) x(t) + 2x^T(t) P_x \bar{B}_{ij} \varphi(t) \\
&\quad + x^T(t) C_i^T C_j x(t) + \varphi^T(t) Z^T Z \varphi(t) \\
&\quad + 2x^T(t) C_i^T Z \varphi(t) - \gamma_2 \varphi^T(t) \varphi(t) \Big) \\
&= \sum_{i=1}^{k}\sum_{j=1}^{k} \mu_i\mu_j \begin{bmatrix} x(t) \\ \varphi(t) \end{bmatrix}^T \Theta_{ij} \begin{bmatrix} x(t) \\ \varphi(t) \end{bmatrix}
\end{aligned}$$

where

$$\Theta_{ij} = \begin{bmatrix} \Upsilon_{ij} & P_x \bar{B}_{ij} + C_i^T Z \\ * & -\gamma_2 I + Z^T Z \end{bmatrix} \tag{70}$$

with $\Upsilon_{ij} = (A_i + B_i\bar{K}_jC_i)^T P_x + P_x(A_i + B_i\bar{K}_jC_i) + C_i^T C_j$.
Thus, $J_x(t) < 0$, if

$$\sum_{i=1}^{k}\sum_{j=1}^{k}\mu_i\mu_j\Theta_{ij} < 0 \tag{71}$$

By applying Schur complement, (70) can be written as:

$$\Theta_{ij} = \begin{bmatrix} \bar{\Upsilon}_{ij} & P_xB_i\bar{K}_jN & P_xM_i & P_xE_i & C^T \\ * & -\gamma_2 I & 0 & 0 & N^T \\ * & * & -\gamma_2 I & 0 & 0 \\ * & * & * & -\gamma_2 I & 0 \\ * & * & * & * & -\gamma_2 I \end{bmatrix} \tag{72}$$

where $\bar{\Upsilon}_{ij} = (A_i + B_i\bar{K}_jC_i)^T P_x + P_x(A_i + B_i\bar{K}_jC_i)$.
Premultiplying and postmultiplying by $\Pi = diag\left\{P_x^{-1}, P_x^{-1}, I, I, I\right\}$ and its transpose in (72), then we obtain

$$\sum_{i=1}^{k}\sum_{j=1}^{k}\mu_i\mu_j\left(\begin{bmatrix} \Omega_{ij} & B_i\bar{K}_jN\bar{P}_x & M_i & N & \bar{P}_xC_i^T \\ * & -\gamma_2\bar{P}_x\bar{P}_x & 0 & 0 & \bar{P}_xN^T \\ * & * & -\gamma_2 I & 0 & 0 \\ * & * & * & -\gamma_2 I & 0 \\ * & * & * & * & -\gamma_2 I \end{bmatrix}\right) < 0 \tag{73}$$

where $\Omega_{ij} = A_i\bar{P}_x + \bar{P}_xA_i + B_i\bar{K}_jC_i\bar{P}_x + \bar{P}_x^TC_i^T\bar{K}_j^TB_i^T$ and $\bar{P}_x = P_x^{-1}$.
Based on Lemma 1, it is easy to obtain that

$$\bar{P}_x + \bar{P}_x \leq \bar{P}_x\bar{P}_x + I \tag{74}$$

From $\gamma_2 > 0$, (74) can be expressed as:

$$-\gamma_2\bar{P}_x\bar{P}_x \leq -2\gamma_2\bar{P}_x + \gamma_2 I \tag{75}$$

Then, (73) can be rewritten as:

$$\sum_{i=1}^{k}\sum_{j=1}^{k}\mu_i\mu_j\Sigma_{ij} < 0 \tag{76}$$

where

$$\Sigma_{ij} = \begin{bmatrix} \Omega_{ij} & B_i\bar{K}_jN\bar{P}_x & M_i & N & \bar{P}_xC_i^T \\ * & -2\gamma_2\bar{P}_x + \gamma_2 I & 0 & 0 & \bar{P}_xN^T \\ * & * & -\gamma_2 I & 0 & 0 \\ * & * & * & -\gamma_2 I & 0 \\ * & * & * & * & -\gamma_2 I \end{bmatrix} \tag{77}$$

Notice that the matrix inequality $\Omega_{ij} < 0$ is a Bilinear Matrix Inequalities (BMIs). Denoting $C_i\bar{P}_x = RC_i$ and $\bar{K}_jR = \bar{S}_j$, so that $K_jC_i\bar{P}_x = \bar{S}_jC_i$. Substituting the result into (77) yields

$$\sum_{i=1}^{k}\sum_{j=1}^{k}\mu_i\mu_j\Xi_{ij} \leq 0 \tag{78}$$

where

$$\Xi_{ij} = \begin{bmatrix} \Phi_{ij} & B_i\bar{K}_jN\bar{P}_x & M_i & N & \bar{P}_xC_i^T \\ * & -2\gamma_2\bar{P}_x + \gamma_2 I & 0 & 0 & \bar{P}_xN^T \\ * & * & -\gamma_2 I & 0 & 0 \\ * & * & * & -\gamma_2 I & 0 \\ * & * & * & * & -\gamma_2 I \end{bmatrix} \tag{79}$$

with

$$\Phi_{ij} = A_i\bar{P}_x + \bar{P}_xA_i + B_iS_jC_i + B_i^TS_j^TC_i^T \tag{80}$$

The gains of controller are given by

$$\bar{K}_i = R^{-1}\bar{S}_i \tag{81}$$

Furthermore, Ξ_{ij} can be further decomposed as below:

$$\Delta_{ij} + UV + (UV)^T < 0 \tag{82}$$

where

$$\Delta_{ij} = \begin{bmatrix} \Phi_{ij} & 0 & M_i & N & \bar{P}_xC_i^T \\ * & -2\gamma_2\bar{P}_x + \gamma_2 I & 0 & 0 & \bar{P}_xN^T \\ * & * & -\gamma_2 I & 0 & 0 \\ * & * & * & -\gamma_2 I & 0 \\ * & * & * & * & -\gamma_2 I \end{bmatrix}$$

$$U = \begin{bmatrix} \bar{K}_j^TB_i^T & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$V = \begin{bmatrix} 0 & N\bar{P}_x & 0 & 0 & 0 \end{bmatrix}$$

By using Lemma 3, it is follows that

$$\Delta_{ij} + UV + (UV)^T \leq \Delta_{ij} + \epsilon^{-1}UU^T + \epsilon V^TV \tag{83}$$

From (83), (78) is equivalent to

$$\sum_{i=1}^{k}\sum_{j=1}^{k}\mu_i\mu_j\Psi_{ij} < 0 \tag{84}$$

where

$$\Psi_{ij} = \begin{bmatrix} \Phi_{ij} & 0 & M_i & N & \bar{P}_xC_i^T & B_i\bar{K}_j & 0 \\ * & -2\gamma_2\bar{P}_x + \gamma_2 I & 0 & 0 & \bar{P}_xN^T & 0 & \epsilon\bar{P}_xC_i \\ * & * & -\gamma_2 I & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma_2 I & 0 & 0 & 0 \\ * & * & * & * & -\gamma_2 I & 0 & 0 \\ * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & -\epsilon I \end{bmatrix}$$

If (84) is verified, we have $J_x(t) < 0$, which can guarantee the closed-loop system is asymptotically stable. □

6 Simulation Example

In this example, The inverted pendulum with cart system [37] is considered:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \dfrac{g\sin(x_1(t)) - mlax_2^2(t)\frac{\sin(2x_1(t))}{2} - ba\cos(x_1(t))x_4(t) - a\cos(x_1(t))(F - f_c)}{\frac{4l}{3} - mla\cos(x_1(t))^2} \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = \dfrac{-mga\frac{\sin(2x_1(t))}{2} + \frac{4mla}{3}x_2^2(t)\sin(x_1(t)) - bax_4(t) + \frac{4a}{3}(F - f_c)}{\frac{4}{3} - ma\cos(x_1(t))^2} \end{cases}$$

where $x_1(t)$ and $x_2(t)$ are the angular position and velocity, respectively; $x_3(t)$ and $x_4(t)$ are the cart position and velocity, respectively;

m is the pendulum mass, M is the cart mass, g is the gravity constant, and $a = 1/(m+M)$. The values of the parameters used in this simulation are $m = 0.2kg$, $M = 0.8kg$, $l = 0.5m$, and $L = 2m$.

Then, we get the TS fuzzy system with the following fuzzy rules:

Rule1: If $x_1(t)$ is about 0, Then

$$\begin{cases} \dot{x}(t) = A_1x(t) + B_1(u(t) + f_a(t)) + E_1d(t) \\ y(t) = C_1x(t) + Nf_s(t) \end{cases}$$

Rule1: If $x_1(t)$ is about $\pm\frac{\pi}{4}$, Then

$$\begin{cases} \dot{x}(t) = A_2x(t) + B_2(u(t) + f_a(t)) + E_2d(t) \\ y(t) = C_2x(t) + Nf_s(t) \end{cases}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{g}{\frac{4l}{3}-mla} & 0 & 0 & \frac{ba}{\frac{4l}{3}-mla} \\ 0 & 0 & 0 & 1 \\ \frac{-mga}{\frac{4}{3}-ma} & 0 & 0 & \frac{-ba}{\frac{4}{3}-ma} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{g\frac{2\sqrt{2}}{\pi}}{\frac{4}{3}-\frac{mla}{2}} & 0 & 0 & \frac{ba\frac{\sqrt{2}}{2}}{\frac{4l}{3}-\frac{mla}{2}} \\ 0 & 0 & 0 & 1 \\ \frac{-mga\frac{2}{\pi}}{\frac{4}{3}-\frac{ma}{2}} & 0 & 0 & -\frac{ba}{\frac{4}{3}-\frac{ma}{2}} \end{bmatrix}$$

$$B_1 = E_1 = \begin{bmatrix} 0 \\ \frac{-a}{\frac{4l}{3}-mla} \\ 0 \\ \frac{\frac{4a}{3}}{\frac{4}{3}-ma} \end{bmatrix}, B_2 = E_2 = \begin{bmatrix} 0 \\ \frac{-a\frac{\sqrt{2}}{2}}{\frac{4l}{3}-\frac{mla}{2}} \\ 0 \\ \frac{\frac{4a}{3}}{\frac{4}{3}-\frac{ma}{2}} \end{bmatrix}$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, N = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$$

we set $d(x,u,t) = -f_c + m\ l\ x_2^2(t)\ \sin(x_1(t))$ where $f_c = \rho\ sign(x_4(t))$ with $\rho = 0.05$. The membership functions for rules 1 and 2 are chosen based on the method of sector nonlinearity [38] as follows:

$$\begin{aligned} \mu_1(x_1(t)) &= \frac{1 - \frac{1}{1+\exp(-14(x_1(t)-\frac{\pi}{8}))}}{1+\exp(-14(x_1(t)+\frac{\pi}{8}))} \\ \mu_2(x_1(t)) &= 1 - \mu_1(x_1(t)) \end{aligned}$$

By Choosing $H_1 = 1$, $H_2 = I_3$ and $H_3 = I_2$, we can solve the optimization problem of Theorem 1 using Matlab LMI Toolbox and we obtain $\gamma_1 = 0.3614$, $A^s_{11,1} = -3.2457$, $A^s_{11,2} = -3.2457$, $P_1 = 1.62$ and

$$P_{01} = \begin{bmatrix} 0.164 & 0.043 & -0.3 \\ 0.033 & 0.109 & -0.454 \\ -0.5 & -0.404 & 11.248 \end{bmatrix}, P_{02} = \begin{bmatrix} 0.463 & 0 \\ 0 & 0.804 \end{bmatrix}$$

The observer gains can be calculated as follows:

$$L_1 = \begin{bmatrix} 5.0126 & 1.0616 \\ 17.4978 & 20.7737 \\ -1.3711 & -7.8575 \end{bmatrix}, L_2 = \begin{bmatrix} 5.0066 & 1.0558 \\ 14.7712 & 20.7980 \\ -1.0564 & -11.1295 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} -2.47 & 0.075 \\ 0.053 & -4.381 \end{bmatrix}, K_2 = \begin{bmatrix} -2.341 & 0.014 \\ 0.042 & -3.862 \end{bmatrix}$$

Solving the optimization problem in Theorem 3 results the following controller gain matrices for a minimum attenuation level $\gamma_2 = 1.6128$:

$$\bar{K}_1 = \begin{bmatrix} -2.7838 & -6.8067 & -11.1030 \end{bmatrix}$$
$$\bar{K}_2 = \begin{bmatrix} -2.1520 & -7.5055 & -11.8265 \end{bmatrix}$$

We simulate the closed-loop system be choosing $\sigma_1 = 10$, $\sigma_2 = 15$, $\delta_1 = 0.01$, $\delta_2 = 0.02$, $l_{a,1} = l_{a,2} = 10$ and $l_{s,1} = l_{s,2} = 12$, initial conditions $x_{10} = \pi/20$, $x_{20} = 0$, $x_{30} = 2$ and $x_{40} = 0$. $f_a(t)$ and $f_s(t)$ are assumed that, respectively

$$f_a(t) = \begin{cases} 0 & t < 6 \\ \sin(\pi(t-6)) & t \geq 6 \end{cases}, \quad f_s(t) = \begin{cases} 0 & t < 8 \\ 0.5 & t \geq 8 \end{cases}$$

The simulation results of this example are given in Figures 1-5. Figures 1 and 2 show that the proposed adaptive SMO can estimate the actuator and sensor faults simultaneously despite the presence of uncertainties.

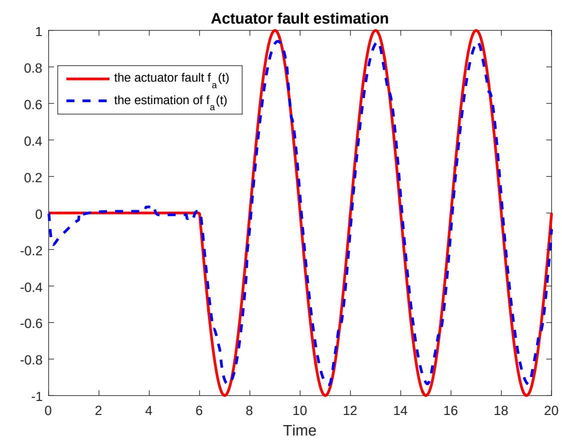

Figure 1: The actuator fault $f_a(t)$ and its estimation.

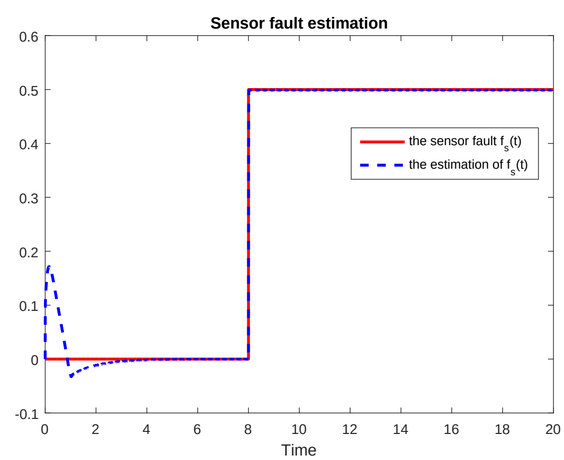

Figure 2: The sensor fault $f_s(t)$ and its estimation.

Simulation results for the systems outputs response are illustrated in Figures 3-5. It is clear to see that the outputs without SOFFTC do not converge to the outputs of the fault-free model (i.e.

without any fault). However, the outputs's trajectories of the system with SOFFTC reach the outputs of nominal model.

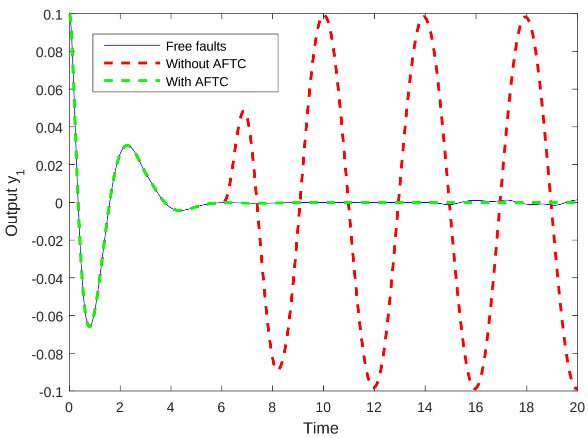

Figure 3: Output $y_1(t)$ under the static output feedback FTC.

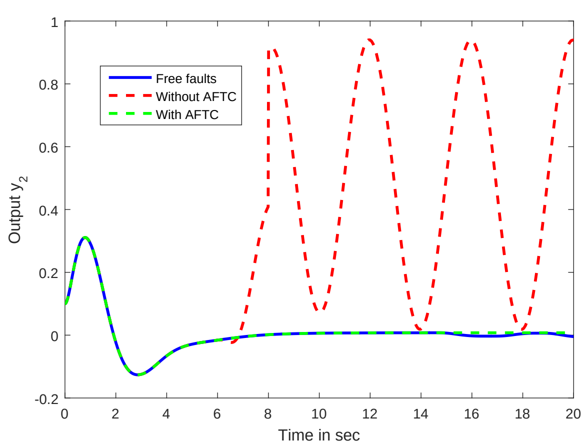

Figure 4: Output $y_2(t)$ under the static output feedback FTC.

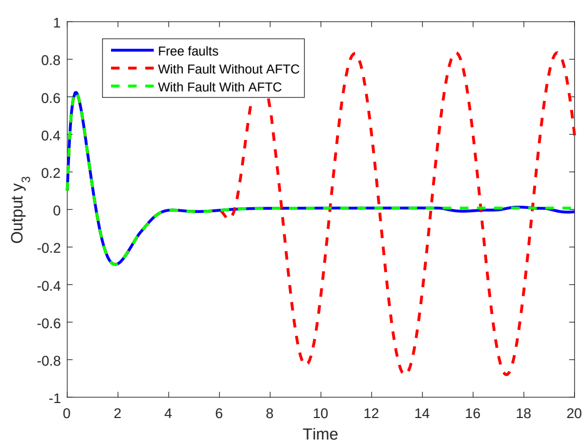

Figure 5: Output $y_3(t)$ under the static output feedback FTC.

7 Conclusion

In this paper, we have considered the problems of FE and FTC for T-S fuzzy systems with uncertainties, actuator and sensor faults, simultaneously. Using the H_∞ optimization technique, an adaptive fuzzy sliding mode observer has been firstly designed to estimate the system state, actuator and sensor faults, simultaneously. Secondly, using the information of online fault estimates, a novel SOFFTC has been developed to compensate the faults and stabilize the closed-loop system. Thus, sufficient condition for the existence of the proposed ASMO and SOFFTC has been formulated in terms of LMIs. Finally, a simulation example was used to show the effectiveness of the proposed methods.

References

[1] S. Ding, "Integrated design of feedback controllers and fault detectors" Annual Reviews in Control, **33**(3), 124-135, 2009. https://doi.org/10.1016/j.arcontrol.2009.08.003

[2] R. Isermann, "Fault-Diagnosis Applications : Model-Based Bondition Monitoring : Actuators, Drives, Machinery, Plants, Sensors, and Fault-Tolerant Systems", Springer Science and Business Media, 2011.

[3] H. Alwi, C. Edwards, C. Tan, "Fault Detection and Fault-Tolerant Control Using Sliding Modes", Springer Science and Business Media, 2011.

[4] S. Dhahri, A. Sellami, F. Ben Hmida, "Robust H_∞ sliding mode observer design for fault estimation in a class of uncertain nonlinear systems with LMI optimization approach" International Journal of Control, Automation and Systems, **10**(5), 1032-1041, 2012. https://doi.org/10.1007/s12555-012-0521-3

[5] M. S. Shaker, R. J. Patton, "Active fault tolerant control for nonlinear systems with simultaneous actuator and sensor faults" International Journal of Control Automation and Systems, **11** (6), 1149-1161, 2013. https://doi.org/10.1007/s12555-013-0227-1

[6] Z. Gao, C. Cecati, S. Ding, "A survey of fault diagnosis and fault tolerant techniques-part i : fault diagnosis with model-based and signal-based approaches" IEEE Transactions on Industrial Electronics, **62**(6), 3757-3767, 2015. https://doi.org/10.1109/TIE.2015.2417501

[7] J. Wang, "$H\infty$ fault-tolerant controller design for networked control systems with time-varying actuator faults" International Journal of Innovative Computing, Information and control, **11**(4), 14711481, 2015.

[8] H. Azmi, M. J. Khosrowjerdi, "Robust adaptive fault tolerant control for a class of lipschitz nonlinear systems with actuator failure and disturbances" Journal of Systems and Control Engineering, **230**(1), 13-22, 2015. https://doi.org/10.1177/0959651815606628

[9] Y. Tian, F. Zhu, "Fault estimation and observer based fault-tolerant controller in finite frequency domain" Transactions of the Institute of Measurement and Control, **40**(5), 1659–1668, 2017. https://doi.org/10.1177/0959651815606628https://doi.org/10.1177/0959651815606628

[10] T. Takagi, M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control" IEEE Transactions on Systems, Man and Cybernetics, **15**(1), 116-132, 1985. https://doi.org/10.1109/TSMC.1985.6313399

[11] K. Zhang, B. Jiang, P. Shi, "Fault estimation observer design for discrete time TakagiSugeno fuzzy systems based on piecewise Lyapunov functions" IEEE Trans. Fuzzy Syst, **20**(1), 192200, 2012. https://doi.org/10.1109/TFUZZ.2011.2168961

[12] Q. Jia, W. Chen, Y. Jin, Y. Zhang, H. Li, "A New Strategy for Fault Estimation in TakagiSugeno Fuzzy Systems via a Fuzzy Learning Observer" in Proceeding of the 11th World Congress on Intelligent Control and Automation (WCICA), Shenyang, China, 3228-3233, 2014. https://doi.org/10.1109/WCICA.2014.7053248

[13] C. Sun, F. Wang, X.Q. He, "Robust fault estimation for a class of TS fuzzy singular systems with time-varying delay via improved delay partitioning approach" J. Control Sci. Eng., **2016**, 2016. https://doi.org/10.1155/2016/6305901.

[14] D. Ding, X. Du, X. Xie, M. Li, "Fault estimation filter design for discretetime TakagiSugeno fuzzy systems" IET Control Theory Appl., **10**(18), 2456-2465, 2016. https://doi.org/10.1049/iet-cta.2016.0318

[15] F. You, S. Cheng, K. Tian, X. Zhang, "Robust fault estimation based on learning observer for TakagiSugeno fuzzy systems with interval timevarying delay" Int. J. Adapt. Control Signal Process., **34**(1), 92–109, 2020. https://doi.org/10.1002/acs.3070

[16] S. Liu, X. Li, H. Wang, J. Yan, "Adaptive fault estimation for T-S fuzzy systems with unmeasurable premise variables" Advances in Difference Equations 2018, **105**(2018), 2018. https://doi.org/10.1186/s13662-018-1571-5

[17] Z. Gao, X. Shi, S.X. Ding, "Fuzzy state/disturbance observer design for TS fuzzy systems with application to sensor fault estimation" IEEE Transactions on Cybernetics, **38**(3), 875-880, 2018. https://doi.org/10.1109/TSMCB.2008.917185

[18] Q. K. Shen, B. Jiang, V. Cocquempot, "Fault tolerant control for TS fuzzy systems with application to nearspace hypersonic vehicle with actuator faults" IEEE Trans. Fuzzy Syst., **20**(4), 652665, 2012. https://doi.org/10.1109/TFUZZ.2011.2181181

[19] X. H. Li, F. L. Zhu, A. Chakrabarty, "Nonfragile fault tolerant fuzzy observer-based controller design for nonlinear systems" IEEE Trans. Fuzzy Syst., **24**(6), 16791689, 2016. https://doi.org/10.1109/TFUZZ.2016.2540070

[20] M. Liu, X. B Cao, and P. Shi, "Fault Estimation and Tolerant Control Fuzzy Stochastic Systems" IEEE Trans. Fuzzy Syst., **21**(2), 221229, 2013.

[21] A. Navarbaf, M. J. Khosrowjerdi, "Fault-tolerant controller design with fault estimation capability for a class of nonlinear systems using generalized Takagi-Sugeno fuzzy model" Transactions of the Institute of Measurement and Control, **41**(15), 42184229, 2019. https://doi.org/10.1177/0142331219853687

[22] M. Liu, X. Cao, P. Shi, "Fuzzy-model-based fault-tolerant design for nonlinear stochastic systems against simultaneous sensor and actuator faults" IEEE Trans. Fuzzy Syst., **21**(5), 789799, 2013. https://doi.org/10.1109/TFUZZ.2012.2224872

[23] M. Sami, R. J. Patton, "Active fault tolerant control for nonlinear systems with simultaneous actuator and sensor faults" Int. J. Control Autom. Syst., **11**(6), 11491161, 2013. https://doi.org/10.1007/s12555-013-0227-1

[24] S. Makni, M. Bouattour, A.E. Hajjaji, M. Chaabane, "Robust fault tolerant control based on adaptive observer for Takagi-Sugeno fuzzy systems with sensor and actuator faults: Application to single-link manipulator" Transactions of the Institute of Measurement and Control, **2020**, 2020. doi.org/10.1177/0142331220909996.

[25] P. Aboutalebi, A. Abbaspour, P. Forouzannezhad, A. Sargolzaei, "A novel sensor fault detection in an unmanned quadrotor based on adaptive neural observer" Journal of Intelligent and Robotic Systems, **90**, 473484, 2018. https://doi.org/10.1007/s10846-017-0690-7

[26] H. Li, Y. Gao, P. Shi, H. K. Lam, "Observer-based fault detection for nonlinear systems with sensor fault and limited communication capacity" IEEE Transactions on Automatic Control, **61**(9), 27452751, 2016. https://doi.org/10.1109/TAC.2015.2503566

[27] A. Valibeygi, A. Toudeshki, K. Vijayaraghavan, " Observer based sensor fault estimation in nonlinear systems" Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, **230**(8), 759777, 2016. https://doi.org/10.1177/0959651816654070

[28] M. Buciakowski, M. Witczak M, V. Puig, et al., "A bounded-error approach to simultaneous state and actuator fault estimation for a class of nonlinear systems" Journal of Process Control, **52**, 1425, 2017. https://doi.org/10.1016/j.jprocont.2017.01.002

[29] M. Bataghva, M. Hashemi, "Adaptive sliding mode synchronisation for fractional-order non-linear systems in the presence of time-varying actuator faults" IET Control Theory Appl., **12**(3), 37783, 2018. https://doi.org/10.1049/iet-cta.2017.0458

[30] H. Schulte, E. Gauterin, "Fault-tolerant control of wind turbines with hydrostatic transmission using TakagiSugeno and sliding mode techniques" Annual Reviews in Control, **40**, 8292, 2015. https://doi.org/10.1016/j.arcontrol.2015.08.003

[31] H. Moodi, M. Farrokhi, "On observer-based controller design for sugeno systems with unmeasurable premise variables" ISA Transactions, **53**(2), 305316, 2014. https://doi.org/10.1016/j.isatra.2013.12.004

[32] T. Dang, W. Wang, L. Luoh, C. Sun, "Adaptive observer design for the uncertain takagisugeno fuzzy system with output disturbance" IET Control Theory Applications, **6**(10), 13511366, 2011. https://doi.org/10.1016/j.isatra.2013.12.004

[33] M. Corless, J. Tu, "State and input estimation for a class of uncertain systems" Automatica, **34**(6), 757764, 1998. https://doi.org/10.1016/S0005-1098(98)00013-2

[34] V.I. Utkin, Sliding modes in control optimization, Springer, Berlin, 1992.

[35] K. Tanaka, H. Wang, Fuzzy control systems design and analysis: a linear matrix inequality approach, Wiley, New York, USA, 2001.

[36] Z. Gao, P. J. Antsaklis, "Stability of the pseudo inverse method for reconfigurable control systems" Int. J. of Control, **53**, 717–729, 1991. https://doi.org/10.1080/00207179108953643

[37] K. Ogata, Modern control engineering, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1997.

[38] M. C. M. Teixeira, S. H. Zak, "Stabilizing controller design for uncertain nonlinear systems using fuzzy models" IEEE Transaction on Fuzzy Systems, **7**(2), 133–142, 1999. https://doi.org/10.1109/91.755395