Performance Analysis of Phased Array and Frequency Diverse Array Radar Ambiguity Functions

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A R T I C L E I N F O

A B S T R A C T

Radar ambiguity function is widely employed to analyze a radar system's time delay (range) resolution, Doppler frequency (range rate) resolution, mainlobe width and sidelobes in both range and Doppler. In this paper, ambiguity functions of frequency diverse array (FDA) which offers range-angle dependent transmit beampattern and Phased array which yields angle transmit beampattern is investigated. The FDA and Phased array radar ambiguity function is derived and evaluate its performance characteristics in terms of time-delay and Doppler frequency resolutions. Performance analysis has also been done in terms of empirical cumulative distribution function (ECDF). Simulation and results have verified the effectiveness of both radar systems by comparing their performance characteristics.

1 Introduction

Phased array radar system has been widely utilized in many modern communications, radar and navigation systems [1, 2, 3]. In a standard phased-array antenna, the elements are fed coherently and phase shifters or time-delayers are used to scan the beam to the desired direction in space [4]. The offered directional gain is beneficial for target detection and tracking weak targets in the antenna looking direction and at the same time suppressing strong sidelobe interferences from other directions. The phased-array antenna beam steering is fixed in an angle for all ranges and thus, the range and angle of targets cannot be directly estimated from its beamforming output due to an inherent range ambiguity [5].

Recently, a flexible array, namely, frequency diverse array (FDA) has been proposed [6, 7, 8]. The most significant FDA difference from its counterpart phased-array is that a small frequency increment, compared to the carrier frequency, is applied between the FDA elements. This frequency increment yields a range-angle-dependent beampattern for which the beam focus direction will be a function of the angle, range and time [9, 10, 11]. FDA has sparked much investigations because of its promising applications. In [12, 13, 14] FDA have been investigated in terms of range-dependent beam with applications in suppressing range ambiguous clutter. FDA for synthetic aperture radar (SAR) high-resolution imaging was reported in [15]. Furthermore, [16, 17] exploited FDA capabilities in range and angle localization of targets.

It is well established that the radar ambiguity function is a useful tool for evaluating performance metrics of radar system, such as time delay (range) and Doppler frequency resolutions and the probabilities of detection and false alarm. Moreover, it helps in selecting appropriate waveforms depending on the required performance parameters.

Therefore, in this perspective, this paper derived and analyzed the performance characteristics of phased array and FDA radar ambiguity functions in time delay and Doppler frequency resolutions. Ambiguity function performance metric is also provided in terms of empirical cumulative distribution function (ECDF). The performance characteristics are compared and FDA radar exhibited better performance results than the phased array radar.

The remaining sections are organized as follows: In Section II, conventional ambiguity function is reviewed. Section III, ambiguity functions for FDA and phased array is derived. Section IV, provides simulation results. Finally, Section V, concludes the paper.

2 Review Of The Conventional Ambiguity Function

In a conventional radar system, the radar ambiguity function (AF) is a common tool for investigating radar signals [18, 19, 20, 21]. The AF is a two dimensional function that describes the output of a matched filter when the input signal of time delay t_d and Doppler frequency f_d relative to

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nominal values for which the matched filter has been designed. The AF can be expressed as

$$|\chi(t_d, f_d)| \triangleq \left| \int_{-\infty}^{\infty} u(t) u^*(t + t_d) \exp\{j2\pi f_d t\} dt \right| \quad (1)$$

where $(\cdot)^*$ denotes the conjugate operator, $u(t)$ is the radar waveform, t_d and f_d are the mismatch in time delay and Doppler frequency, respectively. The ambiguity function plays a great role in determining the t_d and f_d resolutions of radar. The value of this function is maximum at $(t_d, f_d) = (0, 0)$, and it represents the matched filter output without any mismatch. Hence, the narrower the AF around origin, the better the t_d and f_d resolutions. Therefore, the ideal AF would be an impulse at the origin, however, there is no waveform promising such kind of an AF.

3 Frequency Diverse Array And Phased Array Radar Ambiguity Function

First, we will analyzed FDA signal model and its ambiguity function derivation and then followed by phased array radar. FDA elements can be either excited by the same waveform or different waveforms. Consider N elements of FDA radar signal model. Assume the nth element transmitted signal is given by

$$\psi_n(t) = \exp(j2\pi z_{n,1} t) s(t) \quad (2)$$

where $z_{n,1}$ denotes the nth transmit signal pulse carrier frequency and $s(t)$ can be expressed as

$$s(t) \triangleq \begin{cases} 1, & t \in [0, T_\psi] \\ 0, & otherwise \end{cases} \quad (3)$$

with T_ψ denotes the pulse duration. The nth FDA element emits the signal $\psi_n(t)$ with a frequency increment Δf_n. The signals radiated by the nth element can be expressed as

$$\begin{aligned} s_n(t) &= \sqrt{N} \psi_n(t) \exp(-j2\pi f_n t) w_n^* \\ &= \sqrt{N} u_n(t) w_n^* \end{aligned} \quad (4)$$

where $f_n = f_0 + \Delta f_n$, with f_0 denotes the FDA radar carrier frequency, w_n being weights and $u_n(t)$ defined in (5)

$$u_n(t) = \exp(-j2\pi(f_0 - z_{n,1} + \Delta f_n)t) s(t) \quad (5)$$

By following [16], the transmitted signal propagating along (θ, r) in the far-field is given by

$$S_n(t, \theta, r) = \sqrt{N} \sum_{n=1}^{N} w_n^* a_n(t, \theta, r) u_n(t) \quad (6)$$

and $a_n(t, \theta, r)$ can be written as [22]

$$a_n(t, \theta, r) = e^{\left\{ j2\pi \left(\Delta f_n t - \frac{\Delta f_n r}{c} + n \frac{d f_0 \sin\theta}{c} \right) \right\}} \quad (7)$$

Without loss of validity, a single-antenna is employed in the receiver. Assumed the round-trip time delay to a target

in the angle θ and range r is t_d and $w_n = 1$ is used. The received signal is written as

$$\hat{r}_s(t, t_d, f_d, \theta, r) = \sqrt{N} \sum_{n=1}^{N} a_n(t, \theta, r) u_n(t - t_d) \exp(j2\pi f_d t) \quad (8)$$

where f_d denotes the Doppler frequency. When the received signal is matched with the assumed parameters $(t_d', f_d', \theta', r')$, the output of the matched filter is used to define FDA radar ambiguity function as expressed in (9), where the cross-ambiguity function is

$$\chi_{n,n'}(t_d, f_d) \triangleq \int_0^{T_\psi} u_n(t) u_{n'}^*(t + t_d) \exp(j2\pi f_d t) dt \quad (10)$$

The following summarized FDA radar characteristics:

(1) If the frequency increment Δf_n is fixed, the beam direction is a function of range r dependent.

(2) If the range r is fixed, the beam direction is a function of Δf_n dependent.

(3) If the frequency increment Δf_n is not employed ($\Delta f_n = 0$), the corresponding FDA simplified to phased array radar.

(4) As the frequency increment Δf_n influences the beamwidth, a higher resolution may be achieved for the FDA as compared to the phased array radar.

By using the expression of the FDA radar ambiguity function derived in (9), the phased array ambiguity function can also be derived in a similar manner as in (11)

4 Simulation Results And Discussions

For the simulations, we consider a uniform linear FDA and phased array antenna with 16 elements spaced by d. The radar operational frequency is $10GHz$, frequency increment adopted is $30kHz$, $r = 10kHz$ and $\theta = \frac{\pi}{3}$.

Figure 1(a) shows phased array radar ambiguity function when $\theta = \frac{\pi}{3}$ is adopted and Figure 1(b) depicts FDA radar ambiguity function when $r = 10kHz$ and $\theta = \frac{\pi}{3}$ is also adopted. It can be seen in Figure 1(a) that the phased-array radar has range-Doppler coupled and defocused ambiguity function. This implies that the phased-array radar cannot effectively distinguish range-angle dependent targets, while in Figure 1(b), FDA radar ambiguity function accumulates into a spike with relatively low sidelobe peak. It is evident from Figure 1(b) that, FDA radar can effectively distinguish range-angle dependent targets.

Figure (2a) and 2(b) illustrates the ambiguity functions in time delay and Doppler frequency dimensions. It can be observed in Figure 2(b) that by employment of frequency increment in FDA radar system produces a more focused peak which is useful in target localization than the phased array radar in Figure 2(a).

Figure 3(a) and 3(b) shows the comparisons of ambiguity function versus time delay of phased-array and FDA

$$\chi\left(t_d, f_d, \theta, r, t_{d}{}', f_{d}{}', \theta', r'\right) = \int_{-\infty}^{\infty} \hat{r}_s^H\left(t, t_{d}{}', f_{d}{}', \theta', r'\right) \hat{r}_s\left(t, t_d, f_d, \theta, r\right) dt$$

$$= N \cdot \sum_{n=1}^{N} \sum_{n'=1}^{N} \left\{ \exp\left\{ j2\pi \left(\begin{array}{c} T_0\left(\Delta f_n - \Delta f_{n'}\right) - \frac{\left(\Delta f_n - \Delta f_{n'}\right)}{c} \\ + \frac{f_0 d\left((n-1)\sin\theta - (n'-1)\sin\theta'\right)}{c} \end{array} \right) \right\} \chi_{n,n'}\left(t_d, f_d\right) \right\}$$

(9)

$$\chi\left(t_d, f_d, \theta, t_{d}{}', f_{d}{}', \theta'\right) = \int_{-\infty}^{\infty} \hat{r}_s^H\left(t_{d}{}', f_{d}{}', \theta'\right) \hat{r}_s\left(t_d, f_d, \theta,\right) dt$$

$$= N \cdot \sum_{n=1}^{N} \sum_{n'=1}^{N} \left\{ \exp\left\{ j2\pi \left(\frac{f_0 d\left((n-1)\sin\theta - (n'-1)\sin\theta'\right)}{c} \right) \right\} \chi_{n,n'}\left(t_d, f_d\right) \right\}$$
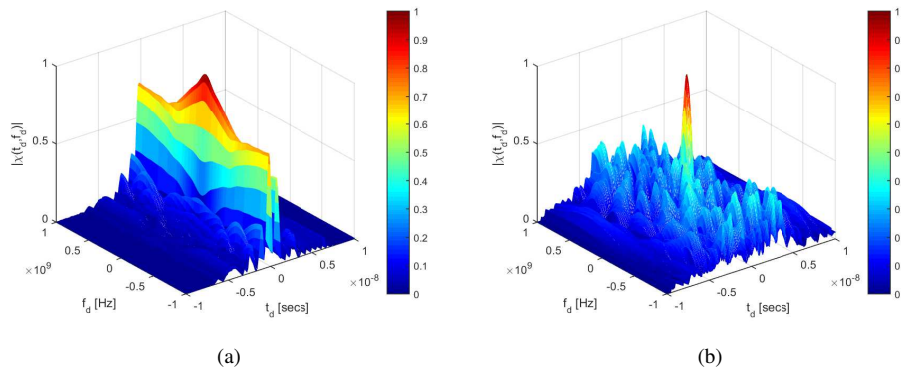
(11)

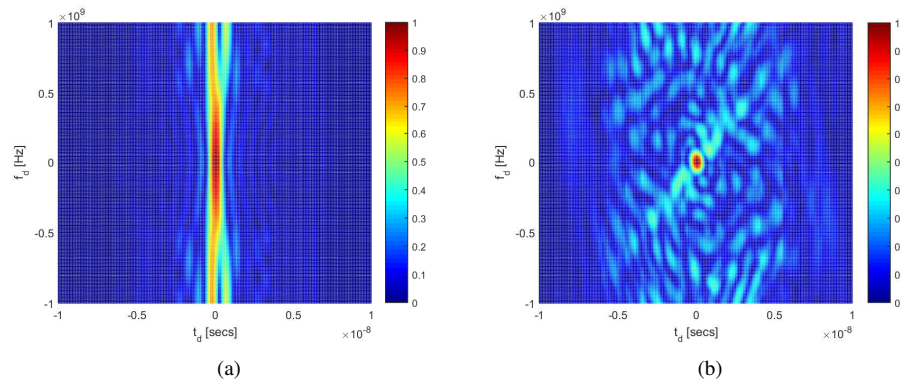Figure 1: Comparisons of 3-D ambiguity functions: a) Phased-array radar, b) FDA radar.

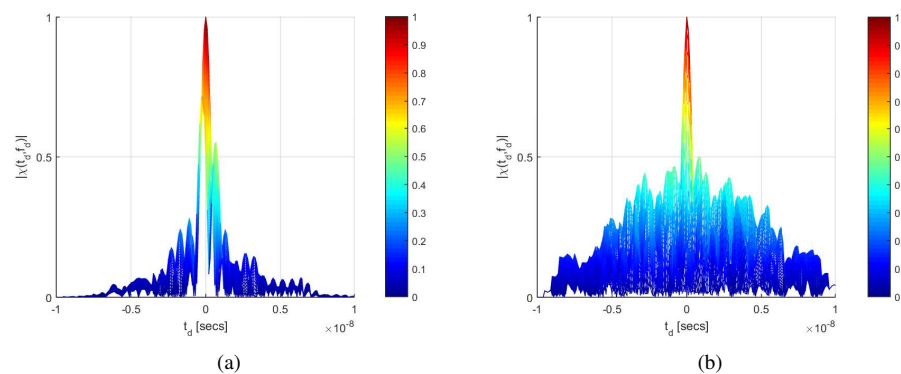Figure 2: Comparisons of ambiguity functions: a) Phased-array radar, b) FDA radar.

Figure 3: Comparisons of ambiguity function versus time delay: a) Phased-array radar, b) FDA radar.
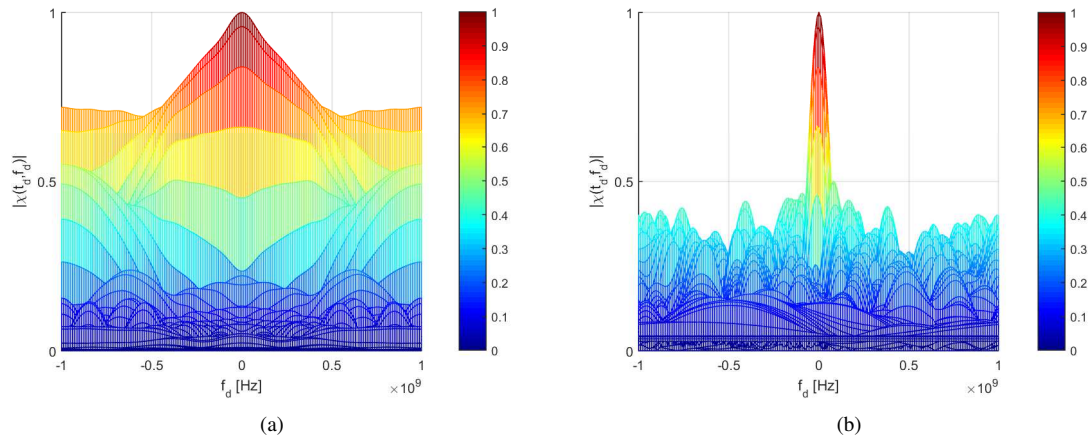
Figure 4: Comparisons of ambiguity function versus Doppler frequency: a) Phased-array radar, b) FDA radar.
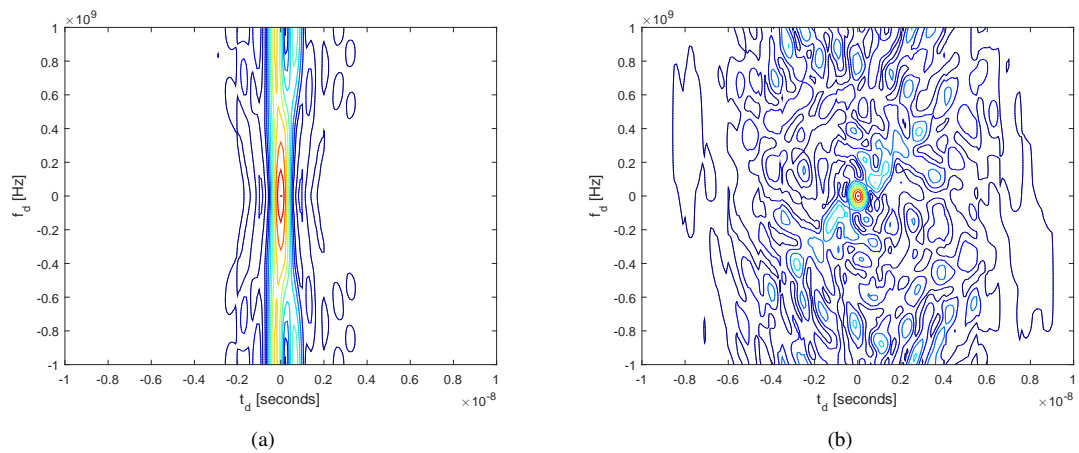

Figure 5: Comparisons of contour plot: a) Phased-array radar, b) FDA radar.

radar, respectively. It can be noticed in Figure 3(a) that the phased-array radar produces narrow beamwidth with maximum sidelobe peak ratio of 0.5510 dB. On the other hand, the FDA radar in Figure 3(b) also achieves a narrow beamwidth with maximum sidelobe peak ratio of 0.5003 dB. Hence, FDA radar outperforms phased array in terms of sidelobe peak ratio.

Figure 4(a) and 4(b) compares the ambiguity function versus Doppler frequency of phased-array and FDA radar, respectively. It can be observed in Figure 4(a) that the phased-array radar produces poor Doppler frequency resolution which is coupled. In contrast, the FDA radar in Figure 4(b) produces a narrow beamwidth Doppler frequency resolution with maximum sidelobe peak ratio of 0.5003 dB. Therefore, FDA radar can estimates Doppler frequency effectively than phased array radar.

Figure 5(a) and 5(b) examines the contour plots of phased-array and FDA radar systems, respectively. It can be seen in Figure 5(b) that the FDA radar contour plot produces a narrow resolution in time delay and Doppler frequency dimensions than the phased array radar in Figure 5(a). It is also evident from Figure 5(b) that the fuzzy area will reduce with increase in N, which would also finally affect the time delay and Doppler frequency resolutions of the system.

Finally, an effective metric used to evaluate the ambiguity function is the empirical cumulative distribution function (ECDF), which represents the percentage of samples of $|\chi(t_d, f_d)|$ lower than a given magnitude. Figure 6 illustrates the ECDF curve to compare both radar systems. The ECDF performance means that the FDA radar yields smaller sidelobe energy than the phased array radar.

5   Conclusion

In this paper, FDA radar and Phased-array radar ambiguity functions has been formulated. Simulation results shows that the phased-array radar has range-Doppler coupled and defocused ambiguity functions. This implies that the phased-array radar cannot effectively distinguish range-angle dependent targets. In contrast, the FDA radar ambiguity obtained lower maximum sidelobe level without degrading the mainlobe performance and can effectively distinguish range-angle dependent targets. In addition, the performance analysis has also been done in terms of empirical cumulative distribution function (ECDF) which proved that FDA radar ambiguity function outperforms the phased array radar ambiguity function. Furthermore, FDA radar ambiguity function produces narrow resolution in time delay and
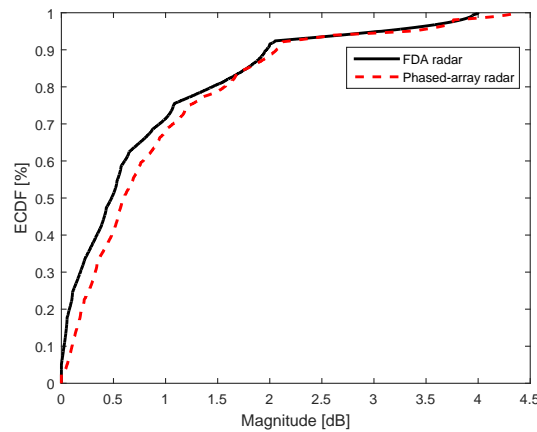
Figure 6: Comparisons of ECDF performances.

Doppler frequency dimensions than its counterpart phased-array radar.

Conflict of Interest   The author declare no conflict of interest.

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