A New Identification Approach of MIMO Hammerstein Model with Separate Nonlinearities

Chekib Ghorbel*, Zeineb Rayouf, Naceur Benhadj Baraiek

Advanced System Laboratory, Polytechnic School of Tunisia BP 743, 2078, La Marsa, Tunisie.

1 Introduction

This paper is an extension of work originally presented in Conference on Science of Electronics, Technologies of Information and Telecommunications (SETIT 2016) [1].

The latter work presented two methods of parametric identification of decoupled multivariable Hammerstein model. It consists of one nonlinear static block and one linear dynamic block. Many process (chemical and biological process, signal processing, etc.) have this structure, for example: pH neutralization processes [2], polymerization reactor [3], distillation columns [4] and dryer process [5].

Many system identification methods have been used to identify the single-input single-output (SISO) model. They can be divided into stochastic methods [6,7], iterative methods [8], over-parameterization methods [9], separable least squares methods [10,11], blind identification methods [12] and frequency domain methods [13].

It is possible to transform the SISO Hammerstein model to MISO and MIMO model which is linear in the parameters [14]. Several approaches have been proposed to identify MIMO Hammerstein model in [15,16,17]. Neuronal networks and fuzzy logic have been used to deal with more general nonlinearities. An approach based on multivariable cardinal cubic spline functions to model the static nonlinearities have been proposed in [18]. The Least Squares Support Vector Machines (LS-SVMs) have been presented in [19,20]. A generalized Hammerstein model consisting of a static polynomial function in series with time-varying linear model is developed in order to model the Hammerstein-like multivariable processes whose linear dynamics vary over the operating space in [21]. In this work, we propose a new coupled structure identification of MIMO model with separate nonlinearities. It is organized as follows: SISO Hammerstein system is presented in part 1 of section 2. A new coupled structure for MIMO Hammerstein system is developed in part 2 of section 2. A quadruple-tank process is given in section 3. Finally, a conclusion is made.

2 Parametric identification of Hammerstein model

2.1 Parametric identification of SISO Hammerstein model

Assume that the Hammerstein model of Figure 1 is composed of a nonlinear block \( F(\cdot) \) associated with a linear sub-system \( \frac{b(q^{-1})}{\lambda(q^{-1})} \). It is described by:

\[
\begin{align*}
y_k &= \frac{b(q^{-1})}{\lambda(q^{-1})} v_k + w_k \\
v_k &= F(u_k)
\end{align*}
\]
with:

\[ A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_n q^{-n_A} \]

\[ B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_m q^{-n_B} \]

\[ v_k = \lambda_1 u_k + \lambda_2 u_k^2 + \ldots + \lambda_N u_k^N \]

\[ q^{-1} \text{ delay operator, } u_k \text{ input of the system, } y_k \text{ output, } \]

\[ v_k \text{ the unmeasurable internal signal and } w_k \text{ represents the}\]

\[ \text{modeling error, external disturbances, etc.} \]

In order to have a unique parameterization of the Hammerstein model structure, the first coefficient of the nonlinear function \( F(.) \) equals to 1, \( \lambda_1 = 1 \) [11,22].

\[
\begin{pmatrix}
  u_k \\
  F(.) \\
  y_k \\
\end{pmatrix} \xrightarrow{B(q^{-1})} \xrightarrow{A(q^{-1})} \xrightarrow{+} \xrightarrow{y_k}
\]

Figure 1: SISO Hammerstein model

The output \( y_k \) is given by:

\[
y_k = -\sum_{i=1}^{n_A} a_i y_{k-i} + \sum_{i=1}^{n_B} b_i \left( u_{k-i} + \sum_{p=2}^{N} \lambda_p u^{p}_{k-i} \right) + w_k
\]

(2)

Equation (2) can be written in the following form:

\[
y_k = \Phi_k^T \theta_k + w_k
\]

(3)

with:

\[
\Phi_k = \begin{pmatrix} Y_k \\
U_k \end{pmatrix}, \quad \theta = \begin{pmatrix} a \\
b \end{pmatrix},
\]

\( \Phi_k \) and \( \theta_k \in \mathbb{R}^{n_R} \) where \( n_R = n_A + N n_B \),

\[
Y_k = \left( -y_{k-1}, -y_{k-2}, \ldots, -y_{k-n_A} \right) \in \mathbb{R}^{n_A},
\]

\[
U_k = \left( U_{1k}, U_{2k}, \ldots, U_{Nk} \right) \in \mathbb{R}^{N n_A},
\]

\[
U_{jk} = \left( u^{1}_{k-1}, u^{2}_{k-2}, \ldots, u^{m}_{k-n_B} \right) \in \mathbb{R}^{n_B},
\]

for \( j = 1, 2, \ldots, N \)

\[
a = \left( a_1, a_2, \ldots, a_{n_A} \right) \in \mathbb{R}^{n_A},
\]

\[
b = \left( b_1, b_2, \ldots, b_{n_B} \right) \in \mathbb{R}^{n_B},
\]

\[
s = \left( \lambda_2 b, \lambda_3 b, \ldots, \lambda_N b \right) \in \mathbb{R}^{N n_B}.
\]

The parameter vector \( \theta \) can be estimated using the RLS algorithm. It is described by the following equations:

\[
\begin{cases}
\hat{\theta}_k = \hat{\theta}_{k-1} + P_k \Phi_k \varepsilon_k \\
\hat{P}_k = \hat{P}_{k-1} - \frac{\hat{P}_{k-1} \Phi_k \Phi_k^T \hat{P}_{k-1}}{1 + \Phi_k^T \hat{P}_{k-1} \Phi_k} \\
\varepsilon_k = y_k - \Phi_k^T \hat{\theta}_k
\end{cases}
\]

with \( P_k \) is the adaptation gain matrix, \( \Phi_k \) is the observation vector and \( \hat{\theta}_k \) is the parameters vector.

### 2.2 Parametric identification of MIMO Hammerstein model

Two structures are used to identify MIMO Hammerstein models in the literature: with separate nonlinearities [21] or with combined nonlinearities [23]. The second case is the most general, but it can cause a very challenging parameter estimation problem because of the large number of parameters to be estimated.

In this paper, we developed a new coupled structure for MIMO Hammerstein model with separate nonlinearities. They are presented in Figure 2 and 4 where \( u_j, v_{i,j}, y_i \) for \( i = 1, 2, \ldots, p \) and \( j = 1, 2, \ldots, m \) are the system input, internal signal and system output. \( F_{i,j}(.) \) are nonlinear function.

#### 2.2.1 A decoupled structure identification of MIMO Hammerstein model

The decoupled structure of MIMO Hammerstein model is given in Figure 2. Each output \( y_{i,k}, i = 1, 2, \ldots, p \) of the multivariable system corresponds to a linear model, which at its input are introduced a nonlinear functions, Figure 3.

![Figure 2: Decoupled structure of MIMO Hammerstein model](image2.png)

![Figure 3: Structure of first model](image3.png)

Each output \( y_{i,k}, i = 1, 2, \ldots, p \), of the MIMO Hammerstein model is proposed as:
\[
A_i(q^{-1})y_{i,k} = \sum_{j=1}^{m} B_{i,j}(q^{-1})v_{j,k}
\]
\[
v_{i,j,k} = F_{i,j}(u_{j,k}) = u_{j,k} + \sum_{\rho=2}^{N} \lambda_{i,j,\rho} u_{j,k}^\rho
\]

with:
\[
A_i(q^{-1}) = 1 + a_{i,1}q^{-1} + a_{i,2}q^{-2} + \ldots + a_{i,n_{Ai}}q^{-n_{Ai}}
\]
\[
B_{i,j}(q^{-1}) = b_{i,j,1}q^{-1} + b_{i,j,2}q^{-2} + \ldots + b_{i,j,n_{Bi,j}}q^{-n_{Bi,j}}
\]

System (5) can be rewritten as:
\[
y_{i,k} = -\frac{n_{Ai}}{q}y_{i,k-\tau} + \sum_{j=1}^{m} \sum_{\rho=2}^{N} B_{i,j}(q^{-1})u_{j,k-\tau}
\]
\[
+ \sum_{j=1}^{m} \sum_{\rho=2}^{N} \sum_{\tau=1}^{\rho} \lambda_{i,j,\rho} u_{j,k-\tau}^\rho
\]

then:
\[
y_{i,k} = -\frac{n_{Ai}}{q}y_{i,k-\tau} + \sum_{j=1}^{m} \sum_{\rho=2}^{N} B_{i,j}(q^{-1})u_{j,k-\tau}
\]
\[
+ \sum_{j=1}^{m} \sum_{\rho=2}^{N} \sum_{\tau=1}^{\rho} S_{i,j,\tau,\rho} u_{j,k-\tau}^\rho
\]

Equation (7) can be written in the following form:
\[
y_{i,k} = \Phi_{i,k}^T \Theta_i
\]

with:
\[
\Phi_{i,k} = \begin{pmatrix} Y_{i,k} \\ U_k \end{pmatrix}, \quad \Theta_i = \begin{pmatrix} A_i \\ B_i \\ S_i \end{pmatrix}
\]
\[
\Phi_{i,k} \text{ and } \Theta_i \in R^{n_R}; \quad n_R = n_{Ai} + \sum_{j=1}^{m} N n_{Bi,j}
\]
\[
Y_{i,k} = \begin{pmatrix} y_{i,k-1} \\ y_{i,k-2} \\ \vdots \\ y_{i,k-n_{Ai}} \end{pmatrix} \in R^{n_{Ai}}
\]
\[
U_k = \begin{pmatrix} U_{1,k} \\ U_{2,k} \\ \vdots \\ U_{m,k} \end{pmatrix} \in R^{mxn_{Bi,j}}
\]
\[
U_{j,k} = \begin{pmatrix} u_{j,k-1} \\ u_{j,k-2} \\ \vdots \\ u_{j,k-n_{Bi,j}} \end{pmatrix} \in R^{n_{Bi,j}}
\]
\[
\Phi_k = \begin{pmatrix} \phi_{1,k} \\ \phi_{2,k} \\ \vdots \\ \phi_{m,k} \end{pmatrix} \in R^{(N-1)xmn_{Bi,j}}
\]
\[
\Phi_j = \begin{pmatrix} \phi_{j,1,k} \\ \phi_{j,2,k} \\ \vdots \\ \phi_{j,n_{Bi,j},k} \end{pmatrix} \in R^{(N-1)xmn_{Bi,j}}
\]
\[
\phi_{j,\tau,k} = \begin{pmatrix} u_{j,k-\tau}^1 \\ u_{j,k-\tau}^2 \\ \vdots \\ u_{j,k-\tau}^N \end{pmatrix} \in R^{N-1}
\]
\[
A_i = \begin{pmatrix} a_{i,1} \\ a_{i,2} \\ \vdots \\ a_{i,n_{Ai}} \end{pmatrix} \in R^{n_{Ai}}
\]
\[
B_i = \begin{pmatrix} B_{i,1} \\ B_{i,2} \\ \vdots \\ B_{i,m} \end{pmatrix} \in R^{mxn_{Bi,j}}
\]
\[
B_{i,j} = \begin{pmatrix} b_{i,j,1} \\ b_{i,j,2} \\ \vdots \\ b_{i,j,n_{Bi,j}} \end{pmatrix} \in R^{n_{Bi,j}}
\]
\[
S_i = \begin{pmatrix} S_{i,1} \\ S_{i,2} \\ \vdots \\ S_{i,m} \end{pmatrix} \in R^{(N-1)xmn_{Bi,j}}
\]
\[
S_{ij} = \begin{pmatrix} S_{i,j,1} \\ S_{i,j,2} \\ \vdots \\ S_{i,j,n_{Bi,j}} \end{pmatrix} \in R^{(N-1)xmn_{Bi,j}}
\]

The steps of the identification scheme are summarized as follows:

1. choosing an initial values for the adaptation matrix,
2. acquiring the input and output of the system and form the vector data as shown in (11) using the present and past values of the input u, output y, and u power,
3. solving the estimate parameter \( A_i, B_i \) and \( S_i \) using the algorithm RLS,
4. solving \( \lambda_{i,j,\rho} \) using the estimated values \( b_{i,j,\tau} \) and \( s_{i,j,\tau,\rho} \) as:
\[
\lambda_{i,j,\rho} = \left( \sum_{\tau=1}^{N_{Bi,j}} b_{i,j,\tau}^2 \right)^{-1} \sum_{\tau=1}^{N_{Bi,j}} s_{i,j,\tau,\rho} b_{i,j,\tau}
\]

2.2.2 A new coupled structure identification of MIMO Hammerstein system

The structure of this method is given in Figure 5. Each output of the system is depended on inputs and all other system outputs, Figure 4.
$A_i(q^{-1}) = 1 + a_{i,1}q^{-1} + a_{i,2}q^{-2} + \ldots + a_{i,n_{Ai}}q^{-n_{Ai}}$
$B_{i,j}(q^{-1}) = b_{i,j,1}q^{-1} + b_{i,j,2}q^{-2} + \ldots + b_{i,j,n_{Bi,j}}q^{-n_{Bi,j}}$
$C_{i,j}(q^{-1}) = c_{i,j,1}q^{-1} + c_{i,j,2}q^{-2} + \ldots + c_{i,j,n_{Ci,j}}q^{-n_{Ci,j}}$

\[y_{i,k} = \sum_{\tau=1}^{n_{Ai}} a_{i,\tau} y_{i,k-\tau} + \sum_{j=1}^{n_{Bi,j}} b_{i,j,\tau} u_{j,k-\tau} + \sum_{j=1}^{n_{Bi,j}} b_{i,j,\tau} \sum_{\rho=2}^{m} \sum_{p=1}^{N} b_{i,j,\tau} l_{i,j,\rho} u_{j,k-\tau} + \sum_{l=1}^{p} \sum_{i=1}^{n_{Ci,l}} c_{i,l,\tau} y_{l,k-\tau}\]

\[(11)\]

\[y_{i,k} = \sum_{\tau=1}^{n_{Ai}} a_{i,\tau} y_{i,k-\tau} + \sum_{j=1}^{n_{Bi,j}} b_{i,j,\tau} u_{j,k-\tau} + \sum_{j=1}^{n_{Bi,j}} b_{i,j,\tau} \sum_{\rho=2}^{m} \sum_{p=1}^{N} b_{i,j,\tau} l_{i,j,\rho} u_{j,k-\tau} + \sum_{l=1}^{p} \sum_{i=1}^{n_{Ci,l}} c_{i,l,\tau} y_{l,k-\tau}\]

\[(12)\]

Equation (12) can be written in the following form:

\[y_{i,k} = \Psi_{i,k}^T \Theta_{i,\text{new}}\]

with:

\[
\Psi_{i,k} = \begin{bmatrix}
Y_{i,k} \\
U_k \\
\varphi_{ik} \\
Y_{L,k}
\end{bmatrix}, \quad \Theta_{i,\text{new}} = \begin{bmatrix}
A_i \\
B_i \\
S_i \\
C_i
\end{bmatrix}
\]

\[\Phi_{i,k} \text{ and } \theta_i \in R^{n_R}; \quad n_R = n_{A_i} + \sum_{j=1}^{m} N_{Bi,j} + n_{Ci,l}(p-1)\]

\[Y_{i,k} = \begin{bmatrix} Y_{1,k}, \ Y_{2,k}, \ldots, Y_{p,k}\end{bmatrix} \in R^{(p-1) \times n_{Ci,j}}\]

\[Y_{i,k} = \begin{bmatrix} -y_{i,k-1}, -y_{i,k-2}, \ldots, -y_{i,k-n_{Ci,j}}\end{bmatrix} \in R^{n_{Ci,j}}\]

\[C_i = \begin{bmatrix} C_{i,1}, C_{i,2}, \ldots, C_{i,p}\end{bmatrix} \in R^{(p-1) \times n_{Ci,j}}\]

\[C_{i,j} = \begin{bmatrix} c_{i,j,1}, c_{i,j,2}, \ldots, c_{i,j,n_{Ci,j}}\end{bmatrix} \in R^{n_{Ci,j}}\]

for \(i = 1, 2, \ldots, p, \ j = 1, 2, \ldots, m, \) and \(\tau = 1, 2, \ldots, n_{Bi,j},\)

\[\text{and } l = 1, 2, \ldots, p, \text{ such as } l \neq i.\]

Parametric identification consists to determine the parameters of the system based on vectors of inputs and outputs, using the recursive least squares algorithm.

### 3 Application: quadruple-tank process

A quadruple-tank process is used to illustrate the performance of the proposed structures of the MIMO Hammerstein model.

#### 3.1 System description

The system setup is a model of a chemical plant fragment. Very often tanks are coupled through pipes and the reactant level and flow has to be controlled. The type of the experiments was performed on the 33-041 Coupled Tanks System of Feedback Instruments [24]. This plant, a variant of the quadruple tank process originally proposed in [25], is a model of a fragment of a chemical plant, Figure 6.

![Figure 6: Plant of four coupled tanks [26]](image)

The line diagram of the reservoir system is shown in Figure 7. The coupled tanks unit consists of four tanks placed on a rig. Another reservoir tank is placed at the bottom. In the reservoir two submersible pumps are placed, which pump the water on command to the tanks. The water flows freely to the bottom tanks through the configurable orifice. The way...
the water flows through the setup can be configured in many ways with manual valves labeled (MVA, MVB, MVC, MVD, MVE, MVF, MVG, MV1, MV2, MV3 and MV4).

The inputs $u_1$ and $u_2$ are shown in Figure 8. They are set to $[0 \ldots +5V]$. The sample time used for all simulations is $T_s = 1s$.

![Figure 7: Line diagram for quadruple-tank process](image)

### 3.2 Mathematical modelling

The quadruple-tank process admits the following nonlinear model [25] which has been assembled in simulink:

\[
\begin{align*}
\dot{h}_1(t) &= \eta v_1(t) - \frac{a_1}{A} \sqrt{2gh_1(t)} - \frac{a_2}{A} \sqrt{2gh_3(t)} \\
\dot{h}_2(t) &= \frac{a_1}{A} \sqrt{2gh_1(t)} - \frac{a_2}{A} \sqrt{2gh_2(t)} \\
\dot{h}_3(t) &= \eta v_2(t) - \frac{a_3}{A} \sqrt{2gh_3(t)} - \frac{a_4}{A} \sqrt{2gh_1(t) - h_3(t)} \\
\dot{h}_4(t) &= \frac{a_3}{A} \sqrt{2gh_3(t)} - \frac{a_4}{A} \sqrt{2gh_4(t)}
\end{align*}
\]

where $h_i$, for $i = 1, 2, 3, 4$, denote the water level in the corresponding tank and $v_i$, for $i = 1, 2$, are voltages applied to the pumps. $a_i$, for $i = 1, 2, 3, 4$, are the outlet area of the tanks, $a_{13}$ is the outlet area between tanks 1 and 3; $\eta$ constant relating the control voltage with the water flow from the pump, $A$ is the cross-sectional area of the tanks and $g$ is the gravitational constant.

### 3.3 Simulation and results

The proposed structures have been tested with the model parameters presented in Table 1.

<table>
<thead>
<tr>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_i$</td>
<td>0–25</td>
<td>Water level of tank i</td>
</tr>
<tr>
<td>$v_i$</td>
<td>0–5</td>
<td>Voltage level of pump i</td>
</tr>
<tr>
<td>$S$</td>
<td>0.014</td>
<td>Cross-sectional area</td>
</tr>
<tr>
<td>$a_i$</td>
<td>5e-5</td>
<td>Outlet area of tank i</td>
</tr>
<tr>
<td>$a_{13}$</td>
<td>5e-5</td>
<td>Outlet area between T1 and T3</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2e-3</td>
<td>Water level of tank i</td>
</tr>
<tr>
<td>$g$</td>
<td>9.81</td>
<td>Gravitational constant</td>
</tr>
</tbody>
</table>

Table 1: Parameters of the plant

![Figure 8: Inputs $u_1$ and $u_2$](image)

![Figure 9: Response of the real and estimated $h_1$](image)

![Figure 10: Response of the real and estimated $h_2$](image)
Figures 9 to 12 show the results of the experiments with two methods from a comparative point of view. Each of them shows a superposition of the actual output and the two outputs estimated as their two error curves. Solid line represents the real output, dotted line represents the estimated systems output signals with the decoupled structure and the dash dot lines represent the estimated systems output signals with the coupled structure in all four graphs. The responses of the original system and the results of the proposed method are very similar. It is clear that the error corresponding to the proposed structure is smaller than that corresponding to the decoupled structure. On inspection, the proposed structure is seen to work better than the decoupled structure. Thus, the feasibility and superiority of this proposed identification method are validated.

4 Conclusion

In this study, a new coupled structure identification of MIMO Hammerstein model with separate nonlinearities has been developed. The method is based on RLS algorithm. A comparative study using simulation results for quadruple-tank process, between the proposed structure and the decoupled structure is discussed. Simulation results reveal the performance and effectiveness of the proposed method.

As a perspective, we will judge the performance of the method used in the presence of perturbations.

References


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