Fault Tolerant Control Based on PID-type Fuzzy Logic Controller for Switched Discrete-time Systems: An Electronic Throttle Valve Application

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1 Introduction

Actuator faults may cause undesired system behaviour and sometimes lead to instability, hence it is necessary to develop Fault Tolerant Control (FTC) methods against actuator faults of uncertain nonlinear systems. In the past decades some FTC design methods have been proposed for several classes of nonlinear systems with actuator faults [1-3]. It consists in computing control laws by taking into account the faults affecting the system in order to maintain acceptable performances and to preserve stability of the system in the faulty situations [4]. From the point of view of FTC strategies, the literature considers two main groups of techniques: passive and active ones. In passive FTC, the faults are treated as uncertainties. Therefore, the control is designed to be robust only to the specified faults [4]. Active FTC techniques consist in adapting the control law using the information given by the Fault Detection and Isolation (FDI) block [5]. The informations issued from the FDI block are used by the FTC module to reconfigure the control law in order to compensate the fault and to ensure an acceptable system performances.

The study of this problem was extended to switched systems in [6-8]. In [7], a switched discrete-time system with state delay has been considered. The design method is based on the construction of a filter and a fault estimation approach. In [8], an adaptive fuzzy tracking control method for a class of switched nonlinear systems with arbitrary switchings and with actuator faults has been proposed. The proposed control scheme guarantee the stability of the whole switched control system based on the common Lyapunov function stability theory and attenuate the effect of the actuator faults on the control performance by designing a new fuzzy controller to accommodate uncertain actuator faults. In [9], an observer has been built to detect the fault when it occurs. The problem of FTC for switched linear systems is addressed by using a nominal control law designed in the absence of any fault, associated with fault detection, localization and reconfiguration techniques to maintain the stability of the system under an arbitrary switching signal in the presence of sensor faults. A state trajectory tracking has been proposed in [4] for actuator faults and observer bank based on controllers with switching mechanism for sensor faults has been also presented in [10]. A nonlinear observer based controller, adopting the so-
called parallel distributed compensation structure, has been designed to choose an adequate state estimate to compensate the effects of the faults on the system in [11]. Switched systems are dynamical systems consisting of a collection of continuous-time subsystems. Switched systems have attracted more attention due to their significance in the modelling of many engineering applications, such as chemical processes, robot manipulators and power systems. Stability analysis and synthesis of switched systems have been made using Lyapunov function to ensure stability of the switched systems such in [12].

In this paper, in order to achieve the FTC for a switched discrete-time systems, a set of PID-type Fuzzy Logic Controller (FLC) is implemented to minimize the error between the desired flat trajectory and the estimated state and to maintain the stability of the system in the presence of actuator faults. The estimated state is provided from an Unknown Input Observer (UIO). Based on residual analysis, a switching strategy using stateflow is then designed. The global stability for the switched systems is studied by Lyapunov theory and expressed as a Linear Matrix Inequalities (LMIs).

The paper is organized as follows. In Section 2, the FTC problem statement is formulated. In Section 3, the fault detection method using UIO is introduced. In Section 4, the PID-type FLC is described. Section 5 deals with the flatness property. The proposed approach is applied to an ETV in Section 6. Finally, the conclusion is drawn in Section 7.

2 FTC problem statement

Consider the discrete time switched system, which can be formulated such that

\[ x(k+1) = A_j x(k) + B_j u(k) + E_j d(k) + B_j f_a(k) \]  \hspace{1cm} (1)

\[ y(k) = C_j x(k) \]

where \( x(k) \in \mathbb{R}^n \) is the state vector, \( u(k) \in \mathbb{R}^p \) the control input, \( y(k) \in \mathbb{R}^q \) the measured output, \( d(k) \in \mathbb{R}^r \) the unknown disturbance input and \( f_a(k) \in \mathbb{R}^p \) the actuator faults. \( A_j \in \mathbb{R}^{n \times n}, \ B_j \in \mathbb{R}^{n \times p}, \ C_j \in \mathbb{R}^{q \times n} \) and \( E_j \in \mathbb{R}^{r \times p} \) are the known constant matrices for \( j \in \psi = \{1, 2, ..., m\} \) and \( m \) the number of models, \( m > 1 \).

In this paper, a FTC structure based on PID-type FLC, given by Figure 1, is proposed to maintain the trajectory tracking and to preserve stability of ETV in the presence of both input disturbance \( d(k) \) and actuator faults \( f_a(k) \).

According to this structure, the FTC approach needs to detect actuator faults firstly and then to design the \( j^{th} \) control law, denoted \( u_j(k) \) given by (2), in order to minimize the error between the desired flat trajectory, generated using the flatness property and the estimated state, provided from the \( j^{th} \) UIO.

\[ u_j(k) = -K_{c,j} \hat{x}_j(k) + y_j^f(k) \]  \hspace{1cm} (2)

\( K_{c,j} \) is the \( j^{th} \) gain control law.
According to Schur complement lemma, the inequalities become

\[
\begin{bmatrix}
-P_j & P_j F_j \\
F_j^T P_j & -P_j
\end{bmatrix} \leq 0, \ \forall j \in \psi
\]

(7)

then the parameters of UIOs can be designed.

**Proof:** Define the state estimation error as \( e(k) = x(k) - \hat{x}(k) \), the dynamic equation can be derived as

\[
e(k + 1) = (A_j - H_j C_j A_j - K_j C_j) e(k)
\]

(8)

with

\[
K_j = K_{j1} + K_{j2}
\]

(9)

In order to make the error decoupled from known control input \( u(k) \), measured output \( y(k) \) and unknown input \( d(k) \), we should let

\[
(H_j C_j - I) E_j = 0
\]

(10)

\[
T_j - (I - H_j C_j) = 0
\]

\[
F_j - (A_j - H_j C_j A_j - K_j C_j) = 0
\]

\[
K_{j2} - (A_j - H_j C_j A_j - K_j C_j) H_j = 0
\]

It can be concluded that

\[
H_j = E_j \left[ (C_j E_j)^T C_j E_j \right]^{-1} (C_j E_j)^T
\]

(11)

\[
A_{j1} = A_j - E_j \left[ (C_j E_j)^T C_j E_j \right]^{-1} (C_j E_j)^T C_j A_j
\]

(12)

\[
F_j = A_j - H_j C_j A_j - K_j C_j = A_{j1} - K_j C_j
\]

(13)

and the error dynamics is given by

\[
e(k + 1) = F_j e(k)
\]

(14)

Equation (14) is UIOs of the switched system (3) if the estimation error tends asymptotically to zero despite the presence of an unknown input \( d(k) \neq 0 \).

Consider the following Lyapunov function

\[
V_j(e(k)) = e(k)^T P_j e(k)
\]

(15)

For \( P_j = P_j^T > 0 \), then, \( V_j(e(k)) > 0 \) holds, the \( \Delta V \) given by (16) is negative

\[
\Delta V_j(e(k)) = e(k)^T F_j^T P_j F_j e(k) - e(k)^T P_j e(k)
\]

(16)

if the following inequalities are satisfied

\[
F_j^T P_j F_j - P_j \leq 0
\]

(17)

Thus, the error system (14) is stable asymptotically. According to Schur complement lemma, the inequalities (17) can be rewritten as

\[
\begin{bmatrix}
-P_j & P_j F_j \\
F_j^T P_j & -P_j
\end{bmatrix} \leq 0
\]

(18)

By substituting \( F_j = A_{j1} - K_j C_j \), the above matrix inequalities become

\[
\begin{bmatrix}
-P_i & P_i (A_{j1} - K_j C_j) \\
(A_{j1} - K_j C_j)^T P_i & -P_i
\end{bmatrix} \leq 0
\]

(19)

and

\[
\begin{bmatrix}
-P_i & P_i (A_j - W_j C_j) \\
(A_j - W_j C_j)^T P_i & -P_i
\end{bmatrix} \leq 0
\]

(20)

for \( W_j = P_j K_j \).

Since \( K_{j1} = P_j^{-1} W_j \), we can obtain the value of \( K_{j1} \) then \( F_j, K_{j2} \) and \( K_j = K_{j1} + K_{j2} \) from \( P_j \) and \( W_j \) solutions of LMIs.

The aim of fault detection is to generate a residual signal \( r_j(k) \), given by (21), which is sensitive to \( f_a(k) \) in the presence of actuator faults which is the purpose of [15].

\[
r_j(k) = y(k) - C_j \hat{x}_j(k)
\]

(21)

## 4 PID-type fuzzy logic controller design

In this study, the FTC approach is based on PID-type FLC. To adjust the input and the output scaling factors of this controller, Genetic Algorithm (GA) optimization technique has been proposed in order to improve the performance of the controller.

### 4.1 PID-type fuzzy logic controller description

We consider a PID-type FLC structure as shown in Figure 2 [16], where \( K_c \in \mathbb{R}^+ \) and \( K_d \in \mathbb{R}^+ \) are the input scaling factors, \( \alpha \in \mathbb{R}^+ \) and \( \beta \in \mathbb{R}^+ \) the output scaling factors.

![Figure 2: The PID-type FLC](image)

The FLC inputs variables, the error \( \epsilon_j(k) \) between the desired flat trajectory \( y_d^j(k) \) and the estimated state \( \hat{x}(k) \), and error variation \( \Delta \epsilon_j(k) \) are given by the equations (22) and (23) where \( T_i \) is the sampling period.

\[
\epsilon_j(k) = -K_{c,j} \hat{x}(k) + y_d^j(k)
\]

(22)

\[
\Delta \epsilon_j(k) = \frac{\epsilon_j(k) - \epsilon_j(k-1)}{T_c}
\]

(23)

The output variable \( \Delta u_j(k) \) of such controller is the variation of the control law signal \( u_j(k) \) which can be defined as (24).

\[
\Delta u_j(k) = \frac{u_j(k) - u_j(k-1)}{T_c}
\]

(24)
The output of the PID-type FLC is given by (25), (17):

\[
\begin{align*}
\dot{u}_j(k) = \alpha \Delta u_j(k) + \beta \int \Delta u_j(k) \, dt \\
= \alpha (Q + K_e u_j(k) + D \Delta u_j(k)) \\
+ \beta \int (Q + K_e u_j(k) + D \Delta u_j(k)) \, dk \\
= \alpha A + \beta A k + (\alpha K_e Q + \beta K_e D) u_j(k) \\
+ \beta K_e \int u_j(k) \, dk + \alpha K_e D \Delta u_j(k)
\end{align*}
\]

where the terms \(Q\) and \(D\) are given by (26) and (27), [10].

\[
\begin{align*}
Q &= \frac{\Delta u_{i(i+1)j} - \Delta u_{ij}}{\varepsilon_{i+1} - \varepsilon_i} \\
D &= \frac{\Delta u_{i(i+1)j} - \Delta u_{ij}}{\varepsilon_{i+1} - \varepsilon_i}
\end{align*}
\]

The fuzzy controllers with product-sum inference method, centroid defuzzification method and triangular uniformly distributed membership functions for the inputs and a crisp output proposed in [18], are used, in our study.

The linguistic levels, assigned to the variables \(\varepsilon_j(k), \Delta \varepsilon_j(k)\) and \(\Delta u_j(k)\), are given in Table 1 as follows: NL: Negative Large; N: Negative; ZR: Zero; P: Positive; PL: Positive Large.

<table>
<thead>
<tr>
<th>(\varepsilon_j(k) / \Delta \varepsilon_j(k))</th>
<th>N</th>
<th>ZR</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>N</td>
<td>ZR</td>
<td>P</td>
</tr>
<tr>
<td>ZR</td>
<td>N</td>
<td>ZR</td>
<td>P</td>
</tr>
<tr>
<td>P</td>
<td>ZR</td>
<td>P</td>
<td>PL</td>
</tr>
</tbody>
</table>

Table 1: Fuzzy rules-base

### 4.2 Optimization of scaling factors using genetic algorithm

To adjust the input and the output scaling factors (\(K_e, K_d\)) and (\(\alpha, \beta\)), GA is used in order to obtain their optimal values.

At first, an initial chromosome population is randomly generated. The chromosomes are candidate solutions to the problem. Then, the fitness values of all chromosomes are evaluated by calculating an objective function. So, based on the fitness of each individual, a group of the best chromosomes is selected through the selection process. The genetic operators, crossover and mutation, are applied to this ‘surviving’ population in order to improve the next generation solutions. Crossover is a recombination operator that combines parts of two parent chromosomes to produce offspring. This operator extracts common features from different chromosomes in order to achieve even better solutions. Mutation is an operator that introduces variations into the chromosome. The modifications can consist of changing one or more values of a chromosome. Through the mutation operator the search space is explored by looking for better points. The process continues until the population converges to the stop criterion.

The most crucial step in applying GA is to choose the objective function that is used to evaluate the fitness of each chromosome. In this paper, the method of tuning PID-type FLC parameters using GA is based on minimizing the Integral of the Squared Error (ISE) used in [18].

### 5 Flatness and trajectory planning

The flatness approach is used in a discrete-time framework for system (1). Let the studied dynamic linear discrete system described by (28)

\[
D_j(q)y_j(k) = N_j(q)v_j(k)
\]

\(q\) is the forward operator, \(v_j(k)\) and \(y_j(k)\) are, respectively, the input and the output signals and \(D_j(q)\) and \(N_j(q)\) the polynomials defined by

\[
\begin{align*}
D_j(q) &= q^n + a_{j,n-1}q^{n-1} + \ldots + a_{j,1}q + a_{j,0} \\
N_j(q) &= b_{j,n-1}q^{n-1} + \ldots + b_{j,1}q + b_{j,0}
\end{align*}
\]

where the parameters \(a_{j,l}\) and \(b_{j,l}\) are constants, \(l = \{0,1,\ldots,n-1\}\).

The system is considered as linear and controllable, consequently it is flat, [16]. The flat output \(z_j(k)\), on which depend the output \(y_j(k)\) and the control \(v_j(k)\), can be seen as being the partial state of a linear system, [19]

\[
\begin{align*}
v_j(k) &= D_j(q)z_j(k) \\
y_j(k) &= N_j(q)z_j(k)
\end{align*}
\]

The open loop control law can be determined by the following relations, [18]

\[
\begin{align*}
u_j^f(k) &= f(z_j^f(k), \ldots, z_j^{(r+1)}(k)) \\
y_j^f(k) &= g(z_j^f(k), \ldots, z_j^{(r+1)}(k))
\end{align*}
\]

\(f\) and \(g\) are vectorial functions and \(z_j^f(k)\) is the desired trajectory that must be differentiable at the \((r+1)\) order.

In order to plan the desired flat trajectory \(z_j^f(k)\), the polynomial interpolation technique is used.

Let consider the state vector: \(Z_j^f(k) = (z_j^f(k), z_j^{(1)}(k), \ldots, z_j^{(r+1)}(k))^T\) containing the desired flat output and its successive derivatives, [18].

The expression of \(Z_j^f(k)\) can be given as following where \(k_0\) and \(k_f\) are two moments known in advance.

\[
Z_j^f(k) = M_{j,1}(k-k_0)c_{j,1}(k_0) + M_{j,2}(k-k_0)c_{j,2}(k_0,k_f)
\]

such that

\[
M_{j,1} = \begin{pmatrix} 1 & k & \ldots & k^{n-1} \\ 0 & 1 & \ldots & k^{n-2} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & 1 \end{pmatrix}
\]

\[
M_{j,2} = \begin{pmatrix} k \cdot k_0 & k \cdot k_0^2 & \ldots & k \cdot k_0^n \\ k_0 & k_0^2 & \ldots & k_0^n \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & 1 \end{pmatrix}
\]
The proposed approach is applied here for the case of electronic throttle valve modeling.

After planning a desired flat trajectory in discrete-time framework, the real output \( y_d(k) \), to be controlled, follows the desired trajectory \( y_d^*(k) \) such that (40), [18].

\[
y_d^*(k) = N_j(q)x_d^*(k)
\]  

6 Application of an electronic throttle valve

6.1 Electronic throttle valve modeling

The electrical part of this system is modeled by (41), [20,21].

\[
u(t) = Ri(t) + L \frac{d}{dt}i(t) + k_v \omega_m(t)
\]  

\( L \) is the inductance, \( R \) the resistance, \( u(t) \) and \( i(t) \) the voltage and the armature current respectively, \( k_v \) a constant electromotive force and \( \omega_m(t) \) the motor rotational speed.

The mechanical part of the throttle is modeled by a gear reducer, characterized by its reduction ratio \( \gamma \) such as (42).

\[
\gamma = \frac{C_g}{C_L}
\]  

\( C_L \) is the load torque and \( C_g \) the gear torque.

The mechanical part is modeled according to (43) and (44), such that, [20,21]

\[
J \frac{d}{dt}\omega_m(t) = C_e - C_f - C_r - C_a
\]  

\[
\frac{d}{dt}\theta(t) = (180/\pi/\gamma)\omega_m(t)
\]

\( \theta(t) \) is the throttle plate angle, \( C_e \) the electrical torque, \( C_f \) the torque caused by mechanical friction, \( C_r \) the spring torque, \( C_a \) the torque generated by the airflow and \( J \) the overall moment of inertia.

The electrical torque is defined by

\[
C_e = k_e i(t)
\]  

where \( k_e \) is a constant.

The electronic throttle valve involves two complex nonlinearities due to \( C_r \) and \( C_f \), given by their static characteristics, [15]

- a dead zone in which the control voltage signal has no effect on the nominal position of the valve plate,
- two hysteresis combined with a saturation, due to the valve plate movement, limited by the maximum and the minimum angles.

The static characteristic of the nonlinear spring torque \( C_r \) is defined by

\[
C_r = \frac{k_r}{\gamma}(\theta - \theta_0) + D \text{sgn}(\theta - \theta_0)
\]  

for \( \theta_{\min} \leq \theta \leq \theta_{\max} \), \( k_r \) is the spring constant, \( D \) a constant, \( \theta_0 \) the default position and \( \text{sgn}(.) \) the following signum function

\[
\text{sgn}(\theta - \theta_0) = \begin{cases} 
1, & \text{if } \theta > \theta_0 \\
-1, & \text{else}
\end{cases}
\]

The friction torque function \( C_f \) of the angular velocity of the throttle plate can be expressed as

\[
C_f = f_v \omega + f_c \text{sgn}(\omega)
\]

where \( f_v \) and \( f_c \) are two constants.

By substituting in equation (13), the expressions \( C_e \), \( C_f \) and \( C_r \) and by neglecting the torque generated by the airflow \( C_a \), the two nonlinearities \( \text{sgn}(\theta - \theta_0) \) and \( \text{sgn}(\omega) \) and the two constants \( k_v, f_v \) and \( k_r, f_c \), the studied system is linear then it can be modeled by the following transfer function \( H(s) \) (49), [21]

\[
H(s) = \frac{(180/\pi/\gamma)k_e}{JLs^2 + JRs^2 + (k_v k_v + L k_v) s + RK_k}
\]

with \( k_v = (180/\pi/\gamma^2)k_r \) and \( s \) the Laplace operator; the identified parameters of this system are given in Table 2 at 25°C, [20].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>2.8</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>( L )</td>
<td>0.0011</td>
<td>( H )</td>
</tr>
<tr>
<td>( k_v )</td>
<td>0.0183</td>
<td>N·m/A</td>
</tr>
<tr>
<td>( k_r )</td>
<td>0.0183</td>
<td>v/rad/s</td>
</tr>
<tr>
<td>( J )</td>
<td>4 × 10⁻⁶</td>
<td>kg·m²</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>16.95</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2: Model’s parameters

Recent work has shown that the ETV can be modeled by two linear models identified from the default position of the throttle plate for two values of the parameter \( k_e \) and for the sampling period \( T_e = 0.002s \), [21].
a model representing the position of the plate above the position by default: \( k_s = 1.877 \times 10^{-4} \text{kg.m}^2 \), then the corresponding discrete-time transfer function \( H_1(q^{-1}) \) given by \([50]\).

\[
H_1(q^{-1}) = \frac{0.007480q^{-1} + 0.01334q^{-2} + 0.0007376q^{-3}}{1 - 1.948q^{-1} + 0.954q^{-2} - 0.006152q^{-3}} \tag{50}
\]

a model representing the position of the plate below the position by default: \( k_s = 1.384 \times 10^{-3} \text{kg.m}^2 \), then the corresponding discrete-time transfer function \( H_2(q^{-1}) \) given by \([51]\).

\[
H_2(q^{-1}) = \frac{0.007497q^{-1} + 0.01333q^{-2} + 0.0007376q^{-3}}{1 - 1.946q^{-1} + 0.954q^{-2} - 0.006152q^{-3}} \tag{51}
\]

### 6.2 Simulation results

In order to test the proposed fault tolerant control approach, two models of an ETV in actuator fault and an unknown disturbance case are given as state space formulation as

\[
x(k + 1) = A_jx(k) + B_1 u(k) + E_j d(k) + B_2 f_\epsilon(k) \tag{52}
\]

\[
y(k) = C_j x(k)
\]

where the parameter matrices and vectors of each model are given as:

\[
A_1 = \begin{bmatrix} 1.948 & -0.954 & 0.006152 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{53}
\]

\[
A_2 = \begin{bmatrix} 1.946 & -0.954 & 0.006152 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{54}
\]

\[
B_1 = B_2 = (1 \ 0 \ 0)^T \tag{55}
\]

\[
C_1 = (0.007480 \ 0.01334 \ 0.0007376) \tag{56}
\]

\[
C_2 = (0.007479 \ 0.01333 \ 0.0007376) \tag{57}
\]

\[
E_1 = E_2 = (1 \ 1 \ 1)^T \tag{58}
\]

The studied system fulfills the conditions of Lemma 1.

\[
\begin{align*}
\text{rank}(C_1 E_1) &= \text{rank}(E_1) = 1 \\
\text{rank}(C_2 E_2) &= \text{rank}(E_2) = 1 
\end{align*} \tag{59}
\]

then \((C_1 A_{11})\) and \((C_2 A_{21})\) are both observable. By applying Theorem, the system is stable asymptotically for any switching signal for symmetric matrix \( P \) definite positive given by \([60]\).

\[
P = e - 010 \begin{bmatrix} 0.1429 & -0.0857 & 0.1903 \\ -0.0857 & 0.5085 & 0.0494 \\ 0.1903 & 0.0494 & 0.4251 \end{bmatrix} \tag{60}
\]

Then, the rest parameters of UIOs are calculated as below

\[
F_1 = \begin{bmatrix} -0.0266 & 0.8566 & -1.5415 \\ -0.0078 & 0.1954 & -0.3515 \\ 0.0165 & -0.4329 & 0.7788 \end{bmatrix} \tag{61}
\]

\[
F_2 = \begin{bmatrix} -0.0233 & 0.8717 & -1.5349 \\ -0.0071 & 0.1998 & -0.3515 \\ 0.0148 & -0.4384 & 0.7715 \end{bmatrix} \tag{62}
\]

\[
T_1 = \begin{bmatrix} 0.9657 & -0.6187 & -0.3470 \\ -0.0343 & 0.3813 & -0.3470 \\ -0.0343 & -0.6187 & 0.6530 \end{bmatrix} \tag{63}
\]

\[
T_2 = \begin{bmatrix} 0.9672 & -0.6196 & -0.3476 \\ -0.0328 & 0.3804 & -0.3476 \\ -0.0328 & -0.6196 & 0.6524 \end{bmatrix} \tag{64}
\]

\[
K_1 = \begin{bmatrix} 0.0002 \\ 0.0031 \\ -0.0056 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0182 \\ 0.0208 \\ -0.0389 \end{bmatrix} \tag{65}
\]

\[
H_1 = \begin{bmatrix} 46.3945 \\ 46.3945 \\ 46.3945 \end{bmatrix}, H_2 = \begin{bmatrix} 46.4707 \\ 46.4707 \\ 46.4707 \end{bmatrix} \tag{66}
\]

In this study, the method of tuning PID-type FLC parameters using GA is based on minimizing the ISE. If \( y_j^d(k) \) is the desired flat trajectory and \( \hat{z}_j(k) \) the estimated state, then we have

\[
\epsilon_j(k) = -K_{\varepsilon,j}\hat{z}_j(k) + y_j^d(k) \tag{67}
\]

For the ISE defined by

\[
ISE = \int_0^t \epsilon_j^2(k) dt \tag{68}
\]

In this paper, the considered fitness function is taken as inverse of this error, i.e. the following performance index

\[
\text{fitness value} = \frac{1}{ISE} \tag{69}
\]

Then, the obtained optimum values of the input/output scaling factors \((K_\varepsilon, K_d, \alpha, \beta)\), using genetic algorithm are given as follows: \( K_\varepsilon = 0.2006, K_d = 0.8071, \alpha = 0.1146 \) and \( \beta = 0.2063 \). The desired discrete time flat trajectory \( z_j^d(k) \), with \( j = \{1, 2\} \) can be computed according to the following polynomial form

\[
z_j^d(k) = \begin{cases} 
& \text{cst1}_{B_1(1)} \text{ if } 0 < k < k_0 \\
& \text{cst2}_{B_1(1)} \text{ if } k_0 < k < k_1 \\
& \text{Poly}_{y_1,j}(k) \text{ if } k_1 < k < k_2 \\
& \text{Poly}_{y_2,j}(k) \text{ if } k_2 < k < k_3 \\
& \text{cst1}_{B_1(1)} \text{ if } k > k_3
\end{cases} \tag{70}
\]

where \( \text{cst1} \) and \( \text{cst2} \) are constant parameters, \( k_0 = 3s, k_1 = 6s, k_2 = 10s \) and \( k_3 = 15s \) are the instants of transitions, \( B_1(1) \) is the static gain between the flat output \( z_j(k) \) and the output signal \( y_j(k) \) for each model and \( \text{Poly}_{y_1,j}(k) \) and \( \text{Poly}_{y_2,j}(k) \) are polynomials calculated using the technique of polynomial interpolation.

The desired trajectories \( y_j^d(k) \) and \( y_j^2(k) \) are then given in Figure 4.
Let us consider the fault vector
\[ f_a(k) = \begin{bmatrix} f_{a1}^T(k) \\ f_{a2}^T(k) \end{bmatrix} \] (71)

such as \( f_{a2} = 0 \) and \( f_{a1} \) defined as follows
\[ f_{a1}(k) = \begin{cases} 0, & k \in [0, 10] \\ -0.1 \sin(0.314k), & k \in [10, 13] \\ 0, & k \in [13, 20] \end{cases} \] (72)

The simulation results illustrated in Figure 5 and Figure 6 show some oscillations for the outputs signals and for the tracking error due to the unknown input. We remark that the system’s responses with and without FTC track desired trajectories with disturbances rejection.

At the time of actuator faults occurrence \( k = 10s \), the system’s behavior was changed. The system without FTC becomes unstable, whereas for the same reference input and by using the FTC, the system remains stable and the tracking error have a small deviation from zero which shows the effectiveness of the proposed FTC approach.

Figure 7, Figure 8, and Figure 9 show the residual values generated using UIOs and the switched signal. The switching between the two models is achieved based on residual values comparison.

7 Conclusion
In this paper, a new fault tolerant control law based on PID-type FLC is designed for the nonlinear complex system ETV, modeled by a multimodel structure. The approach is based on the use of a reference
model generated using the flatness property. The proposed control law is, then, designed to minimize the error between the desired flat trajectory and the estimated state, generated using UIOs, even in the presence of actuator faults based on minimizing the ISE.

References