Short CCA-Secure Attribute-Based Encryption

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ABSTRACT

Chosen-ciphertext attacks (CCA) are typical threat on public-key encryption schemes. We show direct chosen-ciphertext security modification in the case of attribute-based encryption (ABE), where an ABE scheme secure against chosen-plaintext attacks (CPA) is converted into an ABE scheme secure against CCA by individual techniques. Our modification works in the setting that the Diffie-Hellman tuple to be verified in decryption is in the target group of a bilinear map. The employed techniques result in expansion of the secret-key length and the decryption cost by a factor of four, while the public-key and the ciphertext lengths and the encryption cost remain almost the same.

1 Introduction

Access control is one of the fundamental processes and requirements in cybersecurity. Attribute-based encryption (ABE) invented by Sahai and Waters [1], where attributes mean authorized credentials, enables to realize access control which is functionally close to role-based access control (RBAC), but by encryption. In key-policy ABE (KP-ABE) introduced by the subsequent work of Goyal, Pandey, Sahai and Waters [2], a secret key is associated with an access policy over attributes, while a ciphertext is associated with a set of attributes. In a dual manner, in ciphertext-policy ABE (CP-ABE) [2][3][4], a ciphertext is associated with an access policy over attributes, while a secret key is associated with a set of attributes. In a KP-ABE or CP-ABE scheme, a secret key works to decrypt a ciphertext if and only if the associated set of attributes satisfies the associated access policy. The remarkable feature of ABE is attribute privacy; that is, in decryption, no information about the access policy and the identity of the secret key owner in the case of KP-ABE (or, the attributes and the identity of the secret key owner in the case of CP-ABE) leaks except the fact that the set of attributes satisfies the access policy. Since the proposals, it has been studied to attain certain properties such as indistinguishability against chosen-plaintext attacks (IND-CPA) in the standard model [4] and adaptive security against adversary’s choice of a target access policy [5].

In this paper¹ we work through resolving a problem of constructing a shorter ABE scheme that attains indistinguishability against chosen-ciphertext attacks (IND-CCA) in the standard model. Here CCA means that an adversary can collects decryption results of ciphertexts of its choice through adversaries’ attacking. Note that “provable security” of a cryptographic primitive is now a must requirement when we employ the primitive in a system, where it means that an appropriately defined security is polynomially reduced to the hardness of a computational problem. Moreover, the CCA security of an encryption scheme is preferable to attain because the CCA security is one of the theoretically highest securities and hence the scheme can be used widely.

To capture the idea of our approach, let us recall the case of identity-based encryption (IBE). The CHK transformation of Canetti, Halevi and Katz [7] is a generic tool for obtaining IND-CCA secure IBE scheme. It transforms any hierarchical IBE (HIBE) scheme that is selective-ID IND-CPA secure [8] into an IBE scheme that is adaptive-ID IND-CCA secure [8]. A point of the CHK transformation is that it introduces a dummy identity $vk$ that is a verification key of a one-time signature. Then a ciphertext is attached with $vk$ and a signature $\sigma$, which is generated each time one executes encryption. In contrast, the direct chosen-ciphertext security technique for IBE of

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¹This paper is an extension of the work originally presented in SMARTCOMP 2017 [6]. The schemes of KP-ABKEM and KP-ABE have been newly proposed.

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ABKEM: the blinding factor can be considered as a
vk
it introduces a dummy attribute
their transformation is, similar to the case of IBE, that
an ABE scheme that is IND-CCA secure. A point of
or the verifiability [10] that is IND-CPA secure into
transforms any ABE scheme (with the delegatability
scheme individually, the obtained scheme attains bet-
CCA secure. Though the technique needs to treat each
scheme. Their transformation is considered to be an
scheme. To overcome the above obstacle, we employ
 queries. In addition, we also utilize the algebraic trick of Boneh and Boyen
and Kiltz [14] to reply for adversary’s decryption
queries.

1.1 Our Contribution

A contribution is that we fill in the missing piece; we
demonstrate direct chosen-ciphertext security modifi-
cation in the case of the Waters CP-ABE scheme [4]
and the KP-ABE scheme of Ostrovsky, Sahai and Wa-
ters [11]. To overcome the above obstacle, we employ
the technique of the Twin Diffie-Hellman Trapdoor
Test of Cash, Kiltz and Shoup [12]. In addition, we
also utilize the algebraic trick of Boneh and Boyen
[13] and Kiltz [14] to reply for adversary’s decryption
queries.

1.2 Related Works

Waters [4] pointed out that IND-CCA security would
be attained by the CHK transformation. Gorantla,
Boyd and Nieto [15] constructed a IND-CCA secure
CP-ABKEM in the random oracle model. In [10] the
authors proposed a generic transformation of an IND-
CPA secure ABE scheme into a IND-CCA secure ABE
scheme. Their transformation is considered to be an
ABE-version of the CHK transformation, and it is ver-
satile. Especially, it can be applied to non-pairing-
based scheme.

The Waters CP-ABE [4] can be captured as a CP-
ABKEM: the blinding factor can be considered as a
random one-time key. This Waters CP-ABKEM is
IND-CPA secure because the Waters CP-ABE is proved
to be IND-CPA secure. For theoretical simplicity, we
demonstrate an individual conversion of the Waters
CP-ABKEM into a CP-ABKEM which is IND-CCA se-
cure. Then we provide a CP-ABE scheme which is
IND-CCA secure. As for KP-ABE, we demonstrate an
individual conversion of KP-ABKEM of Ostrovsky, Sa-
hai and Waters [11], which is IND-CPA secure, into a
KP-ABKEM which is IND-CCA secure. Then we pro-
vide a KP-ABE scheme which is IND-CCA secure.

Finally, we note that there is a remarkable work of
CP-ABE schemes and KP-ABE schemes with constant-
size ciphertexts [16, 17]. Our direct chosen-ciphertext
security modification is not constant-size ciphertexts
but a different approach for easier implementation in
engineering.

1.3 Organization of the Paper

In Section 2, we survey concepts, definitions and tech-
niques needed. In Section 3, we revisit the concept,
the algorithm and the security of the twin Diffie-
Hellman technique. In Section 4, we construct a CCA-
secure CP-ABKEM from the Waters CPA-secure CP-
ABKEM [4], and provide a security proof. Also, we
describe the encryption version, a CCA-secure CP-
ABE. In Section 5, we construct a CCA-secure KP-
ABKEM from the Ostrovsky-Sahai-Waters CPA-secure
KP-ABKEM [11], and provide a security proof. Also, we
describe the encryption version, a CCA-secure KP-
ABE. In Section 6, we compare efficiency of our CP-
ABE and KP-ABE schemes with the original schemes,
and also, with the schemes obtained by applying the
generic transformation [10] to the original schemes. In Section 7 we conclude our work.

2 Preliminaries

The security parameter is denoted λ. A prime of bit
length λ is denoted p. A multiplicative cyclic group of
order p is denoted G. The ring of exponent domain of
G, which consists of integers from 0 to p − 1 with
modulo p operation, is denoted Zp.

2.1 Bilinear Map

We remark first that our description in the subsequent
sections is in the setting of a symmetric bilinear map
for simplicity, but we can employ an asymmetric bi-
linear map instead for better efficiency as is noted in
Section 6. Let G and GT be two multiplicative cyclic
groups of prime order p. Let g be a generator of G and
e be a bilinear map, e : G×G → GT. The bilinear map
e has the following properties:
1. Bilinearity: for all u, v ∈ G and a, b ∈ Zp, we have
e(aug, vb) = (e(u, v))ab.
2. Non-degeneracy: e(g, g) ̸= idG (·: the identity ele-
ment of the group GT).

Parameters of a bilinear map are generated by a
probabilistic polynomial time (PPT) algorithm Grp on
input λ: (p, G, GT, g, e) ← Grp(λ).

Hereafter we assume that the group operation in
G and GT and the bilinear map e : G×G → GT are
computable in PT in λ.
2.2 Access Structure

Let $\mathcal{U} = \{x_1, \ldots, x_u\}$ be a set of attributes, or simply set $\mathcal{U} = \{1, \ldots, u\}$ by numbering. An access structure, which corresponds to an access policy, is defined as a collection $\mathcal{A}$ of non-empty subsets of $\mathcal{U}$; that is, $\mathcal{A} \subseteq 2^\mathcal{U} \setminus \{\emptyset\}$. An access structure $\mathcal{A}$ is called monotone if for any $B \in \mathcal{A}$ and $B \subset C$, $C \in \mathcal{A}$ holds. The sets in $\mathcal{A}$ are called authorized sets, and the sets not in $\mathcal{A}$ are called unauthorized sets. We will consider in this paper only monotone access structures.

2.3 Linear Secret-Sharing Scheme

We only describe a linear secret-sharing scheme (LSSS) in our context of attribute-based schemes. A secret-sharing scheme $\Pi$ over the attribute universe $\mathcal{U}$ is called linear over $\mathbb{Z}_p$ if:
1. The shares for each attribute form a vector over $\mathbb{Z}_p$.
2. There exists a matrix $\mathcal{A}$ of size $|\mathcal{U}| \times n$ called the share-generating matrix for $\Pi$ and a function $\rho$ which maps each row index $i$ of $\mathcal{A}$ to an attribute in $\mathcal{U} = \{1, \ldots, u\}$: $\rho : \{1, \ldots, u\} \rightarrow \mathcal{U}$.

To make shares, we first choose a random vector $\vec{v} = (v_1, v_2, \ldots, v_u) \in \mathbb{Z}_p^u$; $s$ is a secret to be shared. For $i = 1$ to $u$, we calculate each share $\lambda_i = v_i \cdot M_i$, where $M_i$ denotes the $i$-th row vector of $\mathcal{A}$ and $\cdot$ denotes the inner product. LSSS $\Pi = (M, \rho)$ defines an access structure $\mathcal{A}$ through $\rho$.

Suppose that an attribute set $S$ satisfies $\mathcal{A}$ ($S \in \mathcal{A}$) and let $I_S = \rho^{-1}(S) \subset \{1, \ldots, u\}$. Then, let $\omega_i \in \mathbb{Z}_p$; $i \in I_S$ be a set of constants (linear reconstruction constants) such that if $\lambda_i \in \mathbb{Z}_p$; $i \in I_S$ are valid shares of a secret $s$ according to $M$, then $\sum_{i \in I_S} \omega_i \lambda_i = s$. It is known that these constants $\{\omega_i\}_{i \in I_S}$ can be found in time polynomial in $l$: the row size of the share-generating matrix $M$. If $S$ does not satisfy $\mathcal{A}$ ($S \notin \mathcal{A}$), then no such constants $\{\omega_i\}_{i \in I_S}$ exist.

2.4 Attribute-Based Key Encapsulation Mechanism

Ciphertext-policy attribute-based key encapsulation mechanism (CP-ABKEM). A CP-ABKEM consists of four PPT algorithms (Setup, Encap, KeyGen, Decap) $\Pi_{\text{CP-ABKEM}}$.

Setup$(\lambda, \mathcal{U})$. A setup algorithm Setup takes as input the security parameter $\lambda$ and the attribute universe $\mathcal{U} = \{1, \ldots, u\}$. It returns a public key PK and a master secret key MSK.

Encap$(PK, \mathcal{A})$. An encapsulation algorithm Encap takes as input the public key PK and an access structure $\mathcal{A}$. It returns a random string $\kappa$ and its encapsulation $\psi$. Note that $\mathcal{A}$ is contained in $\psi$.

KeyGen$(PK, MSK, S)$. A key generation algorithm KeyGen takes as input the public key PK, the master secret key MSK and an attribute set $S$. It returns a secret key SK$_S$ corresponding to $S$. Note that $S$ is contained in SK$_S$.

Decap$(PK, SK_S, \psi)$. A decapsulation algorithm Decap takes as input the public key PK, an encapsulation (we also call it a ciphertext according to context) $\psi$ and a secret key SK$_S$. It first checks whether $S \in \mathcal{A}$, where $S$ and $\mathcal{A}$ are contained in SK$_S$ and $\psi$, respectively. If the check result is False, it puts $\kappa = \bot$. It returns a decapsulation result $\hat{\kappa}$.

Chosen-Ciphertext Attack on CP-ABKEM. According to previous works (for example, see [13]), the chosen-ciphertext attack on a CP-ABKEM is formally defined as the indistinguishability game (IND-CCA game). In this paper, we consider the selective game on a target access structure (IND-sel-CCA game); that is, the adversary $\mathcal{A}$ declares a target access structure $\mathcal{A}^\ast$ before $A$ receives a public key PK, which is defined as the following experiment.

Experiment$^\ast$ $\Pi_{\text{CP-ABKEM}}(\lambda, \mathcal{U})$

$\mathcal{A}^\ast \leftarrow \mathcal{A}(\lambda, \mathcal{U})$, $(PK, MSK) \leftarrow \text{Setup}(\lambda, \mathcal{U})$
$\kappa \leftarrow \text{KeyGen}(PK, MSK, \mathcal{A}^\ast)$
$\psi \leftarrow \text{Decap}(PK, SK, \kappa)$

In the above experiment, two kinds of queries are issued by $\mathcal{A}$. One is key-extraction queries. Indicating an attribute set $S_j$, $\mathcal{A}$ queries its key-extraction oracle $\text{KeyGen}(PK, MSK, \cdot)$ for the secret key SK$_S$. Here we do not require any input attribute sets $S_i$ and $S_j$ to be distinct. Another is decapsulation queries. Indicating a pair $(S_j, \psi_j)$ of an attribute set and an encapsulation, $\mathcal{A}$ queries its decapsulation oracle $\text{Decap}(PK, SK, \cdot)$ for the decapsulation result $\hat{\kappa}_j$. Here an access structure $\mathcal{A}_j$ which is used to generate an encapsulation $\psi_j$, is implicitly included in $\psi_j$. In the case that $S \notin \mathcal{A}$, $\hat{\kappa}_j = \bot$ is replied to $\mathcal{A}$. Both kinds of queries are at most $q_k$ and $q_\mathcal{U}$ times in total, respectively, which are polynomial in $\lambda$.

The access structure $\mathcal{A}^\ast$ declared by $\mathcal{A}$ is called a target access structure. Two restrictions are imposed on $\mathcal{A}$ concerning $\mathcal{A}^\ast$. In key-extraction queries, each attribute set $S_i$ must satisfy $S_i \notin \mathcal{A}^\ast$. In decapsulation queries, each pair $(S_j, \psi_j)$ must satisfy $S_j \notin \mathcal{A}^\ast \lor \psi_j \neq \psi^\ast$.

The advantage of the adversary $\mathcal{A}$ over CP-ABKEM in the IND-CCA game is defined as the following probability:

$\text{Adv}_{\mathcal{A}_{\text{CP-ABKEM}}}^\ast(\lambda, \mathcal{U})$ def $\Pr[\text{Experiment}^\ast(\lambda, \mathcal{U}) \text{ returns } \text{Win}]$.

CP-ABKEM is called selectively secure against chosen-ciphertext attacks if, for any PPT adversary $\mathcal{A}$ and for any attribute universe $\mathcal{U}$, $\text{Adv}_{\mathcal{A}_{\text{CP-ABKEM}}}^\ast(\lambda, \mathcal{U})$ is negligible in $\lambda$. Here we must distinguish the two cases: the case that $\mathcal{U}$ is small (i.e. $|\mathcal{U}| = u$ is bounded by a polynomial of $\lambda$) and the case that $\mathcal{U}$ is large (i.e. $u$
is not necessarily bounded by a polynomial of \( \lambda \). We assume the small case in this paper.

In the indistinguishability game against chosen-plaintext attack (IND-CPA game), the adversary \( A \) issues no decapsulation query (that is, \( q_D = 0 \).

**Ciphertext-Policy Attribute-Based Encryption Scheme (CP-ABE).** In the case of the encryption version (i.e., CP-ABE), Encap(\(PK_A \), \(A \)) and Decap(\(PK,SK_s,\psi\)) are replaced by PPT algorithms Encrypt(\(PK,\mathcal{A}\), \(m\)) and Decrypt(\(PK,SK_s,CT\)), respectively, where \( m \) and \( CT \) mean a message and a ciphertext, respectively.

The IND-CCA game for CP-ABE is defined in the same way as for CP-ABKEM above, except the following difference. In Challenge phase, the adversary \( \mathcal{A} \) submits two equal length messages (plaintexts) \( m_0 \) and \( m_1 \). Then the challenger flips a coin \( b = 0 \) or \( 1 \) and gives an encryption result CT of \( \text{Encrypt}(PK, m) \) and \( \text{Decrypt}(PK, CT) \), respectively, where \( m \) and \( CT \) mean a message and a ciphertext, respectively.

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**Key-Policy Attribute-Based Key Encapsulation Mechanism (KP-ABKEM) and Encryption Scheme (KP-ABE).** The key-policy case is analogously defined as the case of the ciphertext-policy case. We state only the syntax and the security experiment of the key-policy ABKEM.

**Setup(\(\lambda,\mathcal{U}\)).** A setup algorithm Setup takes as input the security parameter \( \lambda \) and the attribute universe \( \mathcal{U} = \{1,\ldots,u\} \). It returns a public key \( PK \) and a master secret key \( MSK \).

**Encap(\(PK,SK_A,\psi\)).** A decapsulation algorithm Decap takes as input the public key PK and an attribute set \( S \). It returns a random string \( \kappa \) and its encapsulation \( \psi \). Note that \( S \) is contained in \( \psi \).

**KeyGen(\(PK,SK_A,\mathcal{A}\)).** A key generation algorithm KeyGen takes as input the public key \( PK \), the master secret key \( MSK \) and an access structure \( \mathcal{A} \). It returns a secret key \( SK_A \) corresponding to \( S \). Note that \( A \) is contained in \( SK_A \).

**Chosen-Ciphertext Attack on KP-ABKEM.** The selective game on a target attribute set (IND-sel-CCA game) is defined by the following experiment.

**Experiment_{\text{key-policy ABKEM}}^{\text{IND-sel-CCA}}(\lambda,\mathcal{U})\).** A chosen-ciphertext attack on KP-ABKEM is defined in the same way as for CP-ABKEM above, except the following difference. In Challenge phase, the adversary \( \mathcal{A} \) submits two equal length messages (plaintexts) \( m_0 \) and \( m_1 \). Then the challenger flips a coin \( b = 0 \) or \( 1 \) and gives an encryption result CT of \( \text{Encrypt}(PK, m) \) and \( \text{Decrypt}(PK, CT) \), respectively, where \( m \) and \( CT \) mean a message and a ciphertext, respectively.

**Target Collision Resistant Hash Functions**

Target collision resistant (TCR) hash functions are treated as a family. Let us denote a function family as \( Hfam(\lambda) = \{H_\mu \}_{\mu \in HKey(\lambda)} \). Here \( HKey(\lambda) \) is a hash key space, \( \mu \in HKey(\lambda) \) is a hash key and \( H_\mu \) is a function from \( [0,1]^* \) to \( [0,1]^\lambda \). We may assume that \( H_\mu \) is from \( [0,1]^* \) to \( \mathbb{Z}_p \), where \( p \) is a prime of length \( \lambda \).

Given a PPT algorithm \( CF \), a collision finder, we consider the following experiment (the target collision resistance game).

\[
\text{Experiment}_{\text{CF,Ham}}^{\text{target collision resistant}}(\lambda,\mathcal{U})
\]

\[
m' \leftarrow CF(\lambda,\mu), \quad m \leftarrow HKey(\lambda), \quad m'' \leftarrow H_\mu(m)
\]

if \( m' \neq m \wedge H_\mu(m') = H_\mu(m) \)

then return \( \text{Win} \) else return \( \text{Lose} \).

Then we define \( CF \)'s advantage over \( Hfam \) in the game of target collision resistance as follows.

\[
\text{Adv}_{\text{CF,Ham}}^{\text{target collision resistant}}(\lambda) = \Pr[\text{Experiment}_{\text{CF,Ham}}^{\text{target collision resistant}}(\lambda,\mathcal{U})] - \Pr[\text{Experiment}_{\text{CF,Ham}}^{\text{target collision resistant}}(\lambda,\mathcal{U})]
\]

We say that \( Hfam \) is a TCR function family if, for any PPT algorithm \( CF \), \( \text{Adv}_{\text{CF,Ham}}^{\text{target collision resistant}}(\lambda) \) is negligible in \( \lambda \).

TCR hash function families can be constructed based on the existence of a one-way function \[18\].

**3 The Twin Diffie-Hellman Technique Revisited**

A 6-tuple \((g, X_1, X_2, Y, Z_1, Z_2) \in \mathbb{G}^6\) is called a twin Diffie-Hellman tuple if the tuple is written as \((g, g^{x_1}, g^{y_2}, g^{y_1}, g^{x_2}, g^{x_2})\) for some elements \( x_1, x_2, y \) in \( \mathbb{Z}_p \). In other words, a 6-tuple \((g, X_1, X_2, Y, Z_1, Z_2) \) is a twin Diffie-Hellman tuple (twin DH tuple, for short) if \( Y = g^x \) and \( Z_1 = X_1^y \) and \( Z_2 = X_2^y \).

The following lemma of Cash, Kiltz and Shoup will be used in the security proof to decide whether a tuple is a twin DH tuple or not.

**Lemma 1 (“Trapdoor Test”)** \[12\] Let \( X_1, r, s \) be mutually independent random variables, where \( X_1 \) takes values in \( \mathbb{G} \), and each of \( r, s \) is uniformly distributed over \( \mathbb{Z}_p \).

Define the random variable \( X_2 = X_1^{r + s} \). Suppose that \( Y, Z_1, Z_2 \) are random variables taking values in \( \mathbb{G} \), each of which is defined independently of \( r \). Then the probability that the truth value of \( Z_1^r Z_2 = Y^s \) does not agree with the truth value of \( (g, X_1, X_2, Y, Z_1, Z_2) \) being a twin DH tuple is at most \( 1/p \). Moreover, if \( g, X_1, X_2, Y, Z_1, Z_2 \) is a twin DH tuple, then \( Z_1^r Z_2 = Y^s \) certainly holds.

Note that Lemma \[1\] is a statistical property. Especially, Lemma \[1\] holds without any number theoretic assumption. To be precise, we consider the following experiment of an algorithm \( \text{Cheat} \) with unbounded computational power (not limited to PPT), where \( \text{Cheat} \) is given a tuple \((g, X_1, X_2)\), tries to complete a 6-tuple \((g, X_1, X_2, Y, Z_1, Z_2)\) which satisfies the “Trapdoor Test” but which is not a twin DH tuple.
4.1 Our Construction

Our CP-ABKEM consists of the following four PPT algorithms (Setup, Encap, KeyGen, Decap). Roughly speaking, the Waters original scheme CP-ABKEM_{cpa} (the first scheme in [4]) corresponds to the case $k = 1$ below excluding the "check sum" $(d_1,d_2)$.

**Setup($\lambda$,$\mu$).** Setup takes as input the security parameter $\lambda$ and the attribute universe $\mathcal{U} = \{1, \ldots, u\}$. It runs $\text{Grp}(\lambda)$ to get $(g,G,G_T,e)$, where $G$ and $G_T$ are cyclic groups of order $p$, $e : G \times G \rightarrow G_T$ is a bilinear map and $g$ is a generator of $G$. These become public parameters. Then Setup chooses $u$ random group elements $h_1, \ldots, h_u \in G$ that are associated with the $u$ attributes. In addition, it chooses random exponents $a_k \in \mathbb{Z}_p$, $k = 1, \ldots, 4$, $a \in \mathbb{Z}_p$ and a hash key $\eta \in \text{HKey}(\lambda)$. The public key is published as $\text{PK} = (g,g^a,h_1,\ldots,h_u,e(g,g)^{a_1},\ldots,e(g,g)^{a_4},\eta)$. The authority sets $\text{MSK} = (g^{a_1},\ldots,g^{a_4})$ as the master secret key.

**Encap(PK,$\mathcal{A}$).** The encapsulation algorithm Encap takes as input the public key PK and an LSSS access structure $\mathcal{A} = (M,\rho)$, where $M$ is an $l \times n$ matrix and $\rho$ is the function which maps each row index $i$ of $M$ to an attribute in $\mathcal{U} = \{1, \ldots, u\}$. Encap first chooses a random value $s \in \mathbb{Z}_p$ that is the encryption randomness, and chooses random values $y_2, \ldots, y_l \in \mathbb{Z}_p$. Then Encap forms a vector $\vec{v} = (s,y_2,\ldots,y_l)$. For $i$ from 1 to $l$, it calculates $\lambda_i = \vec{v} \cdot M_i$, where $M_i$ denotes the $i$-th row vector of $M$. In addition, Encap chooses random values $r_1, \ldots, r_l \in \mathbb{Z}_p$. Then, a pair of a random one-time key and its encapsulation $(\kappa,\psi)$ is computed as follows.

- $\psi_{\text{cpa}} = (A,C',((C_i,D_i);i = 1,\ldots,l)), \tau \leftarrow \text{H}_\lambda(\psi_{\text{cpa}});
- \kappa = (1,\psi_{\text{cpa}}, d_1) = \kappa_1 \kappa_3; d_2 = \kappa_2 \kappa_4$.

**KeyGen(MSK, PK, S).** The key generation algorithm KeyGen takes as input the master secret key MSK, the public key PK and a set $S$ of attributes. KeyGen first chooses a random $t_k \in \mathbb{Z}_p, k = 1, \ldots, 4$. It generates the secret key $\text{SK}_S$ as follows.

- $\text{SK}_S = (K_k,L_k, (K_{k,x}; x \in S); k = 1,\ldots, 4)$.

**Decap(PK, $\psi$, $\text{SK}_S$).** The decapsulation algorithm Decap takes as input the public key PK, an encapsulation $\psi$ for an access structure $\mathcal{A} = (M,\rho)$ and a private key $\text{SK}_S$ for an attribute set $S$. It first checks whether $S \in \mathcal{A}$. If the result is FALSE, put $\kappa = \bot$. Otherwise, let $I_S = \rho^{-1}(S) \subset \{1,\ldots,l\}$ and let $\{t_i \in \mathbb{Z}_p; i \in I_S\}$ be a set of linear reconstruction constants. Then, the decapsulation
lution \( \hat{\kappa} \) is computed as follows.

Parse \( \psi \) into \( (\psi_{\text{cpa}} = (A, C', ((C_i, D_i); i = 1, \ldots, l)), d_1, d_2) \);

\( \tau \leftarrow H_{\eta}(\psi_{\text{cpa}}) \);

For \( k = 1 \) to 4:

\[ k'_k = e(C_i, K_{\lambda})/ \prod_{i \in S} e(L_{g', C_i})e(D_i, K_{\lambda}(i))^{\alpha_i} = e(g, g)^{\alpha_S} \]

If \( k'_1 \times k'_3 \neq d_1 \lor k'_2 \times k'_4 \neq d_2 \), then put \( \hat{\kappa} = \bot \), else put \( \hat{\kappa} = k'_1 \).

4.2 Security and Proof

Theorem 1. If the Waters CP-ABKEM_{\text{cpa}} scheme is selectively secure against chosen-plaintext attacks and an employed hash function family \( H \) has target collision resistance, then our CP-ABKEM scheme is selectively secure against chosen-ciphertext attacks.

More precisely, for any given PPT adversary \( A \) that attacks CP-ABKEM in the IND-sel-CCA game where decapsulation queries are at most \( q_A \) times, and for any small attribute universe \( U \), there exist a PPT adversary \( B \) that attacks CP-ABKEM_{\text{cpa}} in the IND-sel-CPA game and a PPT target collision finder \( CF \) on \( H \) that satisfy the following tight reduction.

\[ \text{Adv}^{\text{ind-sel-cca}}_{A, \text{CP-ABKEM}_{\text{cpa}}}(\lambda, U) \leq \text{Adv}^{\text{ind-sel-CPA}}_{B, \text{CP-ABKEM}_{\text{cpa}}}(\lambda, U) + \text{Adv}^{\text{CP}}_{CF, Hfam}(\lambda) + \frac{q_A}{p} \]

Proof. Given any adversary \( A \) that attacks our scheme CP-ABKEM in the IND-sel-CCA game, we construct an adversary \( B \) that attacks the Waters scheme CP-ABKEM_{\text{cpa}} in the IND-sel-CPA game as follows.

Commit to a Target Access Structure. \( B \) is given \( (\lambda, U) \) as inputs, where \( \lambda \) is the security parameter and \( U = \{1, \ldots, u\} \) is the attribute universe. \( B \) invokes \( A \) on input \( (\lambda, U) \) and gets a target access structure \( A' = (M', \tau', p') \) from \( A \), where \( M' \) is of size \( I' \times n' \). \( B \) uses \( A' \) as the target access structure of itself and outputs \( A' \).

Set up. In return to outputting \( A' \), \( B \) receives the public key \( PK_{\text{cpa}} \) for CP-ABKEM_{\text{cpa}}, which consists of the following components.

\[ PK_{\text{cpa}} = (g, g^a, h_1, \ldots, h_u, e(g, g)^{a_S}) \]

To set up a public key \( PK \) for CP-ABKEM, \( B \) hereby needs a challenge instance: \( B \) queries its challenger and gets a challenge instance \( (\hat{\kappa}, \psi'_{\text{cpa}}) \). It consists of the following components.

\[ \hat{\kappa} = e(g, g)^{\rho'} \] OR a random one-time key \( \kappa \in \text{KeySp}(\lambda) \),

\[ \psi'_{\text{cpa}} = ((A', C', g^{\tau'}, ((C_i, D'_i); i = 1, \ldots, I')) \]

Then \( B \) must remain the rest of the parameters of \( PK \) as follows.

Choose \( \eta \leftarrow H_{\kappa}(\psi'_{\text{cpa}}) \) and take \( \tau' \leftarrow H_{\eta}(\psi'_{\text{cpa}}) \);

Put \( e(g, g)^{\rho_1} = e(g, g)^{\rho'} \);

Choose \( \gamma_1, \gamma_2 \leftarrow Z_p, \) put \( e(g, g)^{\rho_2} = e(g, g)^{\gamma_2}/e(g, g)^{\alpha_1 \gamma_1} \);

Choose \( \mu_1, \mu_2 \leftarrow Z_p, \) put \( e(g, g)^{\rho_3} = e(g, g)^{\rho_1}/e(g, g)^{\alpha_1 \tau'} \),

\[ e(g, g)^{\alpha_4} = e(g, g)^{\gamma_2}/e(g, g)^{\alpha_2 \tau'} \].

Note we have implicitly set relations in the exponent domain:

\[ \begin{align*}
    \alpha_2 &= \gamma_2 - \alpha_1 \gamma_1, \\
    \alpha_3 &= \mu_1 - \alpha_1 \tau', \\
    \alpha_4 &= \mu_2 - \alpha_2 \tau' + \gamma_2 - \alpha_1 \gamma_1 \tau'.
\end{align*} \]

(1) A public key PK for CP-ABKEM become:

\[ \text{PK} = (PK_{\text{cpa}}, e(g, g)^{\rho_3}, e(g, g)^{\alpha_3}, e(g, g)^{\alpha_4}, \eta) \]

Then \( B \) inputs PK into \( A \). Note that PK determines the corresponding MSK uniquely.

Phase 1. \( B \) answers for two types of \( A \)'s queries as follows.

(1) Key-Extraction Queries. In the case that \( A \) issues a key-extraction query for an attribute set \( S \subset U \), \( B \) has to simulate \( A \)'s challenger. To do so, \( B \) issues key-extraction queries to \( B \)'s challenger for \( S \) repeatedly up to four times. As replies, \( B \) gets four secret keys of the Waters CP-ABKEM_{\text{cpa}} for a single attribute set \( S \):

\[ \text{SK}_{\text{cpa}, S, k} = (K_{\text{cpa}, k}, L_{\text{cpa}, k}, (K_{\text{cpa}, k, x}; x \in S)), k = 1, \ldots, 4 \]

We remark that, according to the randomness in the key-generation algorithm of the Waters CP-ABKEM_{\text{cpa}}, all four secret keys \( \text{SK}_{\text{cpa}, S, 1}, ..., \text{SK}_{\text{cpa}, S, 4} \) are random and mutually independent. To reply a secret key \( \text{SK}_S \) of our CP-ABKEM to \( A \), \( B \) converts the four secret keys as follows.

\[ \begin{align*}
    K_1 &= K_{\text{cpa}, 1}, \\
    L_1 &= L_{\text{cpa}, 1}, \\
    K_{1, x} &= K_{\text{cpa}, 1, x}, x \in S; \\
    K_2 &= g^{\gamma_1 K_{\text{cpa}, 2}}, \\
    L_2 &= L_{\text{cpa}, 2}, \\
    K_{2, x} &= K_{\text{cpa}, 2, x}, x \in S; \\
    K_3 &= g^{\mu_1 K_{\text{cpa}, 3}}, \\
    L_3 &= L_{\text{cpa}, 3}, \\
    K_{3, x} &= K_{\text{cpa}, 3, x}, x \in S; \\
    K_4 &= g^{\mu_2 - \gamma_2 \tau'} K_{\text{cpa}, 4}, \\
    L_4 &= L_{\text{cpa}, 4}, \\
    K_{4, x} &= K_{\text{cpa}, 4, x}, x \in S.
\end{align*} \]

Then \( B \) replies \( \text{SK}_S = ((K_i, L_i, (K_{i, x}; x \in S)); k = 1, \ldots, 4) \) to \( A \).

(2) Decapsulation Queries. In the case that \( A \) issues a decapsulation query for \( (S, \psi) \), where \( S \subset U \) is an attribute set and \( \psi = (\psi_{\text{cpa}}, d_1, d_2) \) is an encapsulation concerning \( A \), \( B \) has to simulate \( A \)'s challenger. To do so, \( B \) computes the decapsulation result \( \hat{\kappa} \) as follows.

If \( S \notin A \) then put \( \hat{\kappa} = \bot \),

else \( \tau \leftarrow H_{\eta}(\psi_{\text{cpa}}) \);

\[ \begin{align*}
    \hat{y} &= e(C', g)^{\tau'}, \\
    \hat{z}_1 &= d_1/e(C', g)^{\rho_1}, \\
    \hat{z}_2 &= d_2/e(C', g)^{\rho_2}.
\end{align*} \]

If \( \hat{z}_1 \neq \hat{z}_2 \) (call this checking TwinDH-Test) then put \( \hat{\kappa} = k'_1 = \bot \)

else \( \tau = \tau' \) then abort (call this case \text{Abort})

\[ \text{else } \hat{\kappa} = k'_1 = \hat{z}_1^{1/(\tau - \tau')} \]

Challenge. In the case that \( A \) queries its challenger for a challenge instance, \( B \) makes a challenge instance as follows.

\[ \begin{align*}
    d'_1 &= e(C'', g)^{\rho_1}, \\
    d'_2 &= e(C'', g)^{\rho_2}, \\
    \psi' &= (\psi_{\text{cpa}}, d'_1, d'_2).
\end{align*} \]
Then $B$ feeds $(\tilde{x}, \psi')$ to $A$ as a challenge instance. Phase 2. The same as in Phase 1.

**Guess.** In the case that $A$ returns $A$’s guess $\tilde{b}$, $B$ returns $\tilde{b}$ itself as $B$’s guess.

In the above construction of $B$, $B$ can perfectly simulate the real view of $A$ until the case $\text{Abort}$ happens, except for a negligible case, and hence the algorithm $A$ works as designed. To see the perfect simulation with a negligible exceptional case, we are enough to prove the following seven claims.

**Claim 1** The reply $SK_\delta = ((K_\ell, L_\ell, (K_{\ell,x}; x \in S); k = 1, \ldots, 4)$ for a key-extraction query of $A$ is a perfect simulation.

**Proof.** We must consider the implicit relations \[ 1 \]. For the index 2, we have implicitly set the randomness $t_2 = l_{cpa,2}(-\gamma_1)$ and we get:

1. $K_2 = g^{t_2} K_{cpa,2}^{-\gamma_1} = g^{t_2} (g^{a_1} g^{a_2})^{-\gamma_1} = g^{t_2 - a_1} g^{a_2} = g^{a_2} g^{t_2}$,
2. $L_2 = L_{cpa,2}^{-\gamma_1} = g^{t_2}$,
3. $K_{2,x} = K_{cpa,2,x}^{-\gamma_1} = (h_x t_{cpa,2})^{-\gamma_1} = h_x^{-\gamma_1}, x \in S$.

For the index 3 and 4, see Appendix B.

**Claim 2** $(e(g, g), e(g, g)^{a_1}, e(g, g)^{a_2}, \hat{Y}, \hat{Z}_1, \hat{Z}_2)$ is a twin Diffie-Hellman tuple if and only if $(e(g, g), e(g, g)^{a_1}, e(g, g)^{a_2}, e(C', g, d_1, d_2)$ is a twin Diffie-Hellman tuple.

**Proof.** This claim can be proved by a short calculation. See Appendix C.

**Claim 3** If $(e(g, g), e(g, g)^{a_1}, e(g, g)^{a_2}, \hat{Y}, \hat{Z}_1, \hat{Z}_2)$ is a twin Diffie-Hellman tuple, then $(\hat{Y}, \hat{Z}_1, \hat{Z}_2)$ certainly passes the TwinDH-Test: $\hat{Z}_1^{\gamma_1} \hat{Z}_2 = \hat{Y}^{\gamma_2}$.

**Proof.** This claim is a direct consequence of Lemma 1.

**Claim 4** Consider the following event which we name as OVERLOOK:\[ (e(g, g), e(g, g)^{a_1}, e(g, g)^{a_2}, \hat{Y}, \hat{Z}_1, \hat{Z}_2) \]

is NOT a twin DH tuple.

Then, for at most $q_d$ times decapsulation queries of $A$, the probability that at least one OVERLOOK occurs is negligible in $\lambda$. More precisely, the following inequality holds:

\[
\Pr\left[ \bigvee_{i=1}^{q_d} \text{OVERLOOK}_i \right] \leq \frac{q_d}{p}\text{.} \tag{2}
\]

**Proof.** To apply Lemma 2, we construct an algorithm $\text{Cheat}_{\lambda, \delta}$ with unbounded computational power, which takes as input $(e(g, g), e(g, g)^{a_1}, e(g, g)^{a_2})$ and returns $(\hat{Y}, \hat{Z}_1, \hat{Z}_2)$ employing the adversary $A$ as a subroutine.

First, note that the view of $A$ in $\text{Cheat}_{\lambda, \delta}$ is the same as the real view of $A$ and hence the algorithm $A$ works as designed.

Second, note that the return $(\hat{Y}, \hat{Z}_1, \hat{Z}_2)$ of $\text{Cheat}_{\lambda, \delta}$ is randomized in TABLE. Hence:

\[
\text{Adv}_{\text{TwinDH-test}}^{\text{Cheat}_{\lambda, \delta}}(\lambda) = \frac{1}{q_d} \sum_{i=1}^{q_d} \Pr[\text{OVERLOOK}_i]. \tag{3}
\]

Third, applying Lemma 2 to $\text{Cheat}_{\lambda, \delta}$, we get:

\[
\text{Adv}_{\text{TwinDH-test}}^{\text{Cheat}_{\lambda, \delta}}(\lambda) \leq \frac{1}{p}. \tag{4}
\]

Combining (3) and (4), we have:

\[
\Pr\left[ \bigvee_{i=1}^{q_d} \text{OVERLOOK}_i \right] \leq \frac{q_d}{p} \leq \text{Adv}_{\text{TwinDH-test}}^{\text{Cheat}_{\lambda, \delta}}(\lambda). \tag{5}
\]

**Claim 5** The probability that OVERLOOK, never occurs in TwinDH-Test for every $i$ and $\text{Abort}$ occurs is negligible in $\lambda$. More precisely, the following inequality holds:

\[
\Pr\left[ \bigvee_{i=1}^{q_d} \text{OVERLOOK}_i \right] \leq \text{Adv}_{\text{CF}, \text{H}	ext{am}}^{\text{crr}}(\lambda). \tag{6}
\]

**Proof.** This claim is proved by constructing a collision finder $\text{CF}$ on $\text{H}	ext{am}$. See Appendix D.

**Claim 6** The reply $\hat{\ell}$ to $A$ as an answer for a decapsulation query is correct.

**Claim 7** The challenge instance $\psi^* = (\psi_{cpa}^*, d_1^*, d_2^*)$ is correctly distributed.

**Proof.** These two claims are proved by a direct calculation. See Appendices E and F, respectively.

**Evaluation of the Advantage of $B$.** Now we are ready to evaluate the advantage of $B$ in the IND-sel-CPA game. That $A$ wins in the IND-sel-CCA game means that $(\tilde{x}, \psi^* = (\psi_{cpa}^*, d_1^*, d_2^*))$ is correctly guessed. This is equivalent to that $(\tilde{x}, \psi_{cpa}^*)$ is correctly guessed because $\psi_{cpa}^*$ determines the consistent blinding factor $\kappa^* = e(g, g)^{a_2}$ uniquely. This means that $B$ wins in the IND-sel-CPA game.

Therefore, the probability that $B$ wins is equal to the probability that $A$ wins, OVERLOOK never holds in TwinDH-Test for each $i$ and $\text{Abort}$ never occurs. So
we have:

\[ \Pr[B \text{ wins}] = \Pr[(A \text{ wins}) \land \left( \bigwedge_{i=1}^{q_d} \neg \text{Overlook}_i \right) \land \neg \text{Abort}] \]
\[ = \Pr[A \text{ wins}] - \Pr[(A \text{ wins}) \land \neg \left( \bigwedge_{i=1}^{q_d} \neg \text{Overlook}_i \right) \land \neg \text{Abort}] \]
\[ \geq \Pr[A \text{ wins}] - \Pr[\neg \left( \bigwedge_{i=1}^{q_d} \neg \text{Overlook}_i \right) \land \neg \text{Abort}] \]
\[ = \Pr[A \text{ wins}] - \left( \Pr[\bigvee_{i=1}^{q_d} \text{Overlook}_i] + \Pr[\bigwedge_{i=1}^{q_d} \neg \text{Overlook}_i \land \text{Abort}] \right). \]

Substituting (2), (3) and advantages into the above, we have:

\[ \text{Adv}_{\text{CP-ABKEM}}^{\text{ind-sel-cca}}(\lambda, \mu) \geq \text{Adv}_{\text{CP-ABE}}^{\text{ind-sel-cca}}(\lambda, \mu) - \frac{q_d}{p} - \text{Adv}_{\text{CP-ABE}}^{\text{Hfam}}(\lambda). \]

4.3 Encryption Scheme from KEM

It is straightforward to construct our encryption scheme CP-ABE from CP-ABKEM. The IND-sel-CCA security of CP-ABE is proved based on IND-sel-CPA security of the Waters KEM CP-ABKEMKcpa.

\textbf{Setup}(\lambda, \mu). The same as Setup of CP-ABKEM.

\textbf{Encrypt}(PK, A, m). The same as Encap of CP-ABKEM except that Encrypt multiplies m by the blinding factor \( \kappa \) in the group \( G_g \). Encrypt returns CT = (C, m\kappa, \psi = (\psi_{\text{cpa}}, d_1, d_2)).

\textbf{KeyGen}(MSK, PK, S). The same as KeyGen of CP-ABKEM.

\textbf{Decrypt}(PK, CT, SK_s). The same as Decap of CP-ABKEM except that Decrypt divides out C by the decapsulated blinding factor \( \kappa \). Decrypt returns the result \( \tilde{m} \).

4.4 Security and Proof

\textbf{Theorem 2} If the Waters CP-ABKEMKcpa is selectively secure against chosen-plaintext attacks and an employed hash function family Hfam has target collision resistance, then our CP-ABE is selectively secure against chosen-ciphertext attacks. More precisely, for any given PPT ad-
versary \( A \) that attacks CP-ABE in the IND-selected-CCA game where decryption queries are at most \( q_0 \) times, and for any small attribute universe \( \mathcal{U} \), there exist a PPT adversary \( B \) that attacks CP-ABKEM _{c_{\text{cpa}}} in the IND-selected-CPA game and a PPT target collision finder \( CF \) on \( Hfam \) that satisfy the following inequality.

\[
\text{Adv}^{\text{ind-selected-cca}}_{\mathcal{A}, \text{CP-ABE}}(\lambda, \mathcal{U}) \\
\leq 2\left(\text{Adv}^{\text{ind-selected-cpa}}_{\mathcal{B}, \text{CP-ABKEM}_{c_{\text{cpa}}}}(\lambda, \mathcal{U}) + \text{Adv}^\text{ter}_{\mathcal{C}, \text{Hfam}}(\lambda) + \frac{q_d}{p}\right).
\]

**Proof.** Given any adversary \( \mathcal{A} \) that attacks our scheme CP-ABE in the IND-selected-CCA game, we construct an adversary \( \mathcal{B} \) that attacks the Waters KEM CP-ABKEM _{c_{\text{cpa}}} in the IND-selected-CPA game as follows.

**Commit to a Target Access Structure.** The same as that of CP-ABKEM.

**Set up.** In return to outputting \( \mathcal{A}' \), \( \mathcal{B} \) receives the public key \( \text{PK}_\text{c_{\text{cpa}}} \) for CP-ABKEM _{c_{\text{cpa}}}. To set up a public key \( \text{PK} \) for CP-ABE, \( \mathcal{B} \) **herein** needs a challenge instance: \( \mathcal{B} \) queries its challenger and gets a challenge instance \( (\kappa, \psi_{\text{c_{\text{cpa}}}}) \). The rest of procedure is the same as that of CP-ABKEM, and \( \mathcal{B} \) inputs PK into \( \mathcal{A} \).

**Phase 1.** The same as that of CP-ABKEM except that \( \mathcal{B} \) replies a decrypted message \( \hat{m} \) to \( \mathcal{A} \) for a decryption query.

**Challenge.** In the case that \( \mathcal{A} \) submits two plaintexts \( (m_0', m_1') \) of equal length, \( \mathcal{B} \) makes a challenge ciphertext \( \text{CT}' \) as follows and feeds \( \text{CT}' \) to \( \mathcal{A} \).

- Choose \( \psi^* \leftarrow \{0, 1\} \), put \( C^* = m_0', \psi^*; \)
- Put \( d_1' = e(C'^*, g)\mu_1, d_2' = e(C'^*, g)\mu_2; \)
- Put \( \text{CT}' = (C^*, \psi^*; (\psi_{\text{c_{\text{cpa}}}}, d_1', d_2')) \).

**Phase 2.** The same as in Phase 1.

**Guess.** In the case that \( \mathcal{A} \) returns \( \mathcal{A}' \)’s guess \( \hat{b}, \mathcal{B} \) returns \( \hat{b} \) as \( \mathcal{B}' \)’s guess.

**Evaluation of the Advantage of \( \mathcal{B} \).** A standard argument deduces a loss of tightness by a factor of 1/2. That is,

\[
\text{Adv}^{\text{ind-selected-cpa}}_{\mathcal{B}, \text{CP-ABKEM}_{c_{\text{cpa}}}}(\lambda, \mathcal{U}) \\
\geq \frac{1}{2} \text{Adv}^{\text{ind-selected-cca}}_{\mathcal{A}, \text{CP-ABE}}(\lambda, \mathcal{U}) - \frac{q_d}{p} - \text{Adv}^\text{ter}_{\mathcal{C}, \text{Hfam}}(\lambda).
\]

5. **Securing the Ostrovsky-Sahai-Waters KP-ABKEM against Chosen-Ciphertext Attacks**

In this section, we describe our direct chosen-ciphertext security modification by applying to the Ostrovsky-Sahai-Waters KP-ABE [11].

**Overview of Our Modification** The Ostrovsky-Sahai-Waters KP-ABE is proved to be secure in the IND-selected-CPA game [11]. We convert it into a scheme that is secure in the IND-selected-CCA game by employing the Twin Diffie-Hellman technique of Cash, Kiltz and Shoup [12] and the algebraic trick of Boneh and Boyen [13] and Kiltz [14].

In encryption, a ciphertext becomes to contain additional two elements \( (d_1, d_2) \), which function in decryption as a “check sum” to verify that a tuple is certainly a twin DH tuple.

In security proof, the Twin Diffie-Hellman Trapdoor Test does the function instead. It is noteworthy that we are unable to use the bilinear map instead because the tuple to be verified is in the target group. In addition, the algebraic trick enables to answer for adversary’s decryption queries. Note also that both techniques become compatible by introducing random variables.

**Key Encapsulation and Encryption.** The Ostrovsky-Sahai-Waters KP-ABE can be captured as a KP-ABKEM: the blinding factor of the form \( e(g_1, g_2)\mu_1 \) in the Ostrovsky-Sahai-Waters KP-ABE can be considered as a random one-time key. So we call it the Ostrovsky-Sahai-Waters KP-ABKEM hereafter and denote it as KP-ABKEM _{c_{\text{cpa}}}_. Likewise, we distinguish parameters and algorithms of KP-ABKEM _{c_{\text{cpa}}} by the index _{c_{\text{cpa}}}_. For theoretical simplicity, we first develop a KEM KP-ABKEM.

5.1 **Our Construction**

Our KP-ABKEM consists of the following four PPT algorithms (Setup, Encap, KeyGen, Decap). Roughly speaking, the Ostrovsky-Sahai-Waters original scheme KP-ABKEM _{c_{\text{cpa}}} (the first scheme in [11]) corresponds to the case \( k = 1 \) below excluding the “check sum” \( (d_1, d_2) \).

**Setup(\lambda, \mathcal{U})**. Setup takes as input the security parameter \( \lambda \) and the attribute universe \( \mathcal{U} = \{1, ..., \rho\} \). It runs \( \text{Grp}(\lambda) \) to get \( (p, G, G_T, g, \psi) \), where \( G \) and \( G_T \) are cyclic groups of order \( p \), \( e : G \to G_T \) is a bilinear map and \( g \) is a generator of \( G \). These become public parameters. Then Setup chooses \( u \) random group elements \( h_1, ..., h_\rho \in G \) that are associated with the \( u \) attributes. In addition, it chooses random exponents \( \alpha_k \in Z_p, k = 1, ..., 4, a \in Z_p \) and a hash key \( \eta \in H\text{Key}(\lambda) \). The public key is published as \( \text{PK} = (g, g^\psi, h_1, ..., h_\rho, e(g, g)^{\alpha_1}, ..., e(g, g)^{\alpha_4}, \eta) \). The authority sets MSK = \( (\alpha_1, ..., \alpha_4) \) as the master secret key.

**Encap(\text{PK}, S)**. The encapsulation algorithm Encap takes as input the public key PK and a set \( S \) of attributes. Encap first chooses a random value \( s \in Z_p \) that is the encryption randomness. Then, a pair of a random one-time key and its encapsulation \( (\kappa, \psi) \) is computed as follows.

- Put \( C' = g^\psi; \) for \( x \in S \), \( C_x = h_x^\psi \), \( \psi_{\text{c_{\text{cpa}}}} = (S, C', (C_x; x \in S)), \tau \leftarrow H_\psi(\psi_{\text{c_{\text{cpa}}}}); \)
- For \( k = 1 \) to \( 4 : \) \( \kappa_k \leftarrow e(g, g)^{\alpha_1}; d_1 = \kappa_1^\psi \kappa_3, d_2 = \kappa_2^\psi \kappa_4; (\kappa, \psi) = (\kappa_1, (\psi_{\text{c_{\text{cpa}}}}, d_1, d_2)). \)

**KeyGen(MSK, PK, A)**. The key generation algorithm KeyGen takes as input the master secret key MSK, the public key PK and an LSSS access structure \( \mathcal{A} = (M, \rho) \), where \( M \) is an \( I \times \rho \) matrix and \( \rho \) is the function which maps each row index \( i \) of \( M \) to an attribute in \( \mathcal{U} = \{1, ..., \rho\} \). For \( k = 1 \) to \( 4 \), KeyGen first chooses
random values $y_{k,2}, \ldots , y_{k,n} \in \mathbb{Z}_p$ and forms a vector $\vec{v}_k = (\alpha_k, y_{k,2}, \ldots , y_{k,n})$. Then, for $i = 1$ to $l$, it calculates $\lambda_{k,i} = \vec{v}_k \cdot M_i$, where $M_i$ denotes the $i$-th row vector of $M$, and it chooses random values $r_{k,i} \in \mathbb{Z}_p$. KeyGen generates the secret key $SK_A$ as follows.

For $k = 1$ to 4: For $l = 1$ to $l$:
\[
K_{k,i} = g^{\alpha_{k,i}} h_{p(i)}^{r_{k,i}}, \quad L_{k,i} = g^{r_{k,i}}
\]
\[
SK_A = (((K_{k,i}, L_{k,i}; i = 1, \ldots , l); k = 1, \ldots , 4).
\]

Decap($PK, \psi, SK_A$). The decapsulation algorithm Decap takes as input the public key $PK$, an encapsulation $\psi$ for an attribute set $S$ and a private key $SK_A$ for an access structure $A = (M, \rho)$. It first checks whether $S \in A$. If the result is False, put $\hat{\kappa} = \bot$. Otherwise, let $I_\psi = p^{-1}(S) \subset \{1, \ldots , l\}$ and let $\{\psi_i \in \mathbb{Z}_p^2; i \in I_\psi\}$ be a set of linear reconstruction constants. Then, the decapsulation $\hat{\kappa}$ is computed as follows.

Parse $\psi$ into $(\psi_{cpa} = (S, C', (C_x; x \in S)), d_1, d_2)$;
\[
\tau \leftarrow H_{t_\psi}(\psi_{cpa});
\]
For $k = 1$ to 4:
\[
\kappa_k = \prod_{i \in I_\psi} (e(C_x', K_{k,i}/E(L_{k,i}, C_{x(i)})))^{\psi_i} = e(g, g)^{\psi_{cpa}}
\]
If $\kappa_1^{\tau} \kappa_2 = d_1 \vee \kappa_2^{\tau} \kappa_4 = d_2$,
then put $\hat{\kappa} = \bot$, else put $\hat{\kappa} = \kappa_1$.

5.2 Security and Proof

Theorem 3 If the Ostrovsky-Sahai-Waters KP-ABKEM is selectively secure against chosen-plaintext attacks and an employed hash function family $H_fam$ has target collision resistance, then our KP-ABE is selectively secure against chosen-ciphertext attacks. More precisely, for any given PPT adversary $A$ that attacks KP-ABE in the IND-sel-CCA game where decapsulation queries are at most $q_d$ times, and for any small attribute universe $U$, there exist a PPT adversary $B$ that attacks KP-ABKEM in the IND-sel-CPA game and a PPT target collision finder $CF$ on $H_fam$ that satisfy the following tight reduction.

\[
Adv_{\text{ind-sel-cca}}^{\text{KP-ABKEM}}(\lambda, U) \\
\leq Adv_{\text{ind-sel-cpa}}^{\text{KP-ABE}}(\lambda, U) + Adv_{\text{CCA}, H_fam}^{\text{tcr}}(\lambda) + \frac{q_d}{p}.
\]

Proof. We will omit the description of the proof because the proof goes analogously to the case of CP-ABE in Section 5.4.

5.3 Encryption Scheme from KEM

It is straightforward to construct our encryption scheme KP-ABE from KP-ABKEM. The IND-sel-CCA security of KP-ABE is proved based on IND-sel-CPA security of the Waters KEM KP-ABKEM.

Setup($PK, A, m$). The same as Setup of KP-ABKEM.

Encrypt($PK, A, m$). The same as Encap of KP-ABKEM except that Encrypt multiplies $m$ by the blinding factor $\kappa$ in the group $G_T$. Encrypt returns CT = $(C = mk, \psi = (\psi_{cpa}, d_1, d_2))$. The same as KeyGen of KP-ABKEM.

Decrypt($PK, CT, SK_A$). The same as Decap of KP-ABKEM except that Decrypt divides out $C$ by the decapsulated blinding factor $\hat{\kappa}$. Decrypt returns the result $\hat{m}$.

5.4 Security and Proof

Theorem 4 If the Ostrovsky-Sahai-Waters KP-ABKEM is selectively secure against chosen-plaintext attacks and an employed hash function family $H_fam$ has target collision resistance, then our KP-ABE is selectively secure against chosen-ciphertext attacks. More precisely, for any given PPT adversary $A$ that attacks KP-ABE in the IND-sel-CCA game where decapsulation queries are at most $q_d$ times, and for any small attribute universe $U$, there exist a PPT adversary $B$ that attacks KP-ABKEM in the IND-sel-CPA game and a PPT target collision finder $CF$ on $H_fam$ that satisfy the following inequality.

\[
Adv_{\text{ind-sel-cca}}^{\text{KP-ABKEM}}(\lambda, U) \\
\leq Adv_{\text{ind-sel-cpa}}^{\text{KP-ABE}}(\lambda, U) + Adv_{\text{CCA}, H_fam}^{\text{tcr}}(\lambda) + \frac{q_d}{p}.
\]

Proof. We will omit the description of the proof because the proof goes in the same way as the case of CP-ABE in Section 5.4.

6 Efficiency Discussion

First of all, we remark that our individual modification to attain CCA security is applicable when a Diffie-Hellman tuple to be verified is in the target group of a bilinear map $\epsilon: G \times G \rightarrow G_T$. Especially, it is applicable even when an original CPA secure scheme is based on asymmetric pairing \cite{19}, $\epsilon: G_1 \times G_2 \rightarrow G_T$. For example, the Type 3 version \cite{19} of the Waters CP-ABE scheme \cite{4} can be found in \cite{20}. Detailed discussions and results on real implementations are found for the case of CPA-secure ABE schemes \cite{21, 20}. We note here that the efficiency comparison below enables to guess the implementation results of CCA-secure ABE schemes via our modification.

We compare the efficiency of our CP-ABE with the original Waters CP-ABE, and our KP-ABE with the original Ostrovsky-Sahai-Waters KP-ABE. We also compare the efficiency of our schemes with the CCA-secure CP-ABE and KP-ABE schemes obtained by the generic transformation in \cite{10}. Here the generic transformation \cite{10} is considered in the case of a small attribute universe, the delegation type \cite{10} and the Lamport one-time signature \cite{22}. Table 1 shows these comparison. Note that a hash function is applied to compare the $e$ Hellman tuple to be verified is in the target group of a bilinear map $\epsilon$. However, even when an original CPA secure scheme is based on asymmetric pairing, our scheme is selectively secure against chosen-ciphertext attacks.
Table 1: Efficiency comparison of IND-sel-CCA secure ABEs ([10] and ours) with the original IND-sel-CPA secure ABEs [4,11].

<table>
<thead>
<tr>
<th>Scheme</th>
<th>L(PK)</th>
<th>L(SK_σ)</th>
<th>L(CT)</th>
<th>C(Enc)</th>
<th>C(Dec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generic transform [10], CP-ABE</td>
<td>+4λ^2(CT)</td>
<td>+4λ^2(CT)</td>
<td>+3λ^2(bit)</td>
<td>+2λ^2(exp(G))</td>
<td>+2λ^2(pair(e))</td>
</tr>
<tr>
<td>Our individual modification (CP-ABE)</td>
<td>+3(G_7)</td>
<td>×4</td>
<td>+2(G_7)</td>
<td>+4exp(G_7)</td>
<td>×4</td>
</tr>
<tr>
<td>Generic transform [10], KP-ABE</td>
<td>+4λ^2(CT)</td>
<td>+0</td>
<td>+3λ^2(bit)</td>
<td>+2λ^2(exp(G))</td>
<td>+2λ^2(pair(e))</td>
</tr>
<tr>
<td>Our individual modification (KP-ABE)</td>
<td>+3λ^2(CT)</td>
<td>×4</td>
<td>+2(G_7)</td>
<td>+4exp(G_7)</td>
<td>×4</td>
</tr>
</tbody>
</table>

1) λ is the security parameter. (For instance, λ = 224 or 256.)
2) L(data) denotes the length of the data. C(algorithm) denotes the computational amount of the algorithm.
3) + and × mean the increment and the multiplier to the length or to the computational amount of the Waters CP-ABE(kr) and the Ostrovsky-Sahai-Waters KP-ABE(kr).
4) (G), (G_7) and (bit) mean that the lengths are evaluated in the number of elements in G, elements in G_7 and bits, respectively.
5) exp(G) and pair(e) mean the computational amount of one exponentiation in G and one pairing computation by the map e, respectively.

7 Conclusion

We demonstrated direct chosen-ciphertext security modification for ABE in the standard model in the case of the Waters scheme (CP-ABE(kr)_cpa, CP-ABE(kr)_cpa) and the Ostrovsky-Sahai-Waters scheme (KP-ABE(kr)_cpa, KP-ABE(kr)_cpa). We utilized the twin Diffie-Hellman Trapdoor Test of Cash, Kiltz and Shoup and the algebraic trick of Boneh and Boyen [13] and Kiltz [14]. Our modification worked for the setting that the Diffie-Hellman tuple to be verified in decryption was in the target group of the bilinear map. We compared the efficiency of our CCA-secure ABE schemes with the original CPA-secure ABE schemes and with the CCA-secure ABE schemes obtained by the versatile generic transformation.

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References


Appendix

A Proof of Lemma

The only one point to be complemented to the original proof (in [12]) is that even for any algorithm A with unbounded computational power, the statement holds. This is because, conditioning on the input fixed values (g, X₁, X₂), A only reduces the two-dimensional freedom (r, s) ∈ ℤ₂² into the one-dimensional freedom r ∈ ℤ₂ even if A correctly guesses the relation s = rX₁ + sX₂.

B Proof of Claim

For the index 3, we have implicitly set t₃ = Lᶜpa₃(r₃) and we get:

K₃ = gᵖ⁽³⁾Kᶜpa₃(r₃) = gᵖ⁽³⁾(gᵖ⁽¹⁾gᵖ⁽³⁾(X₁)⁻¹) = gᵖ⁽¹⁾⁻¹r₃ = gᵖ⁽³⁾α₃, r₃ = Lᶜpa₃(r₃) = (gᵖ⁽¹⁾α₃)⁻¹r₃ = gᵖ⁽¹⁾⁻¹r₃, x ∈ S.

For the index 4, we have implicitly set t₄ = Lᶜpa₄(r₄) and we get:

K₄ = gᵖ⁽²⁾⁻¹r₄(gᵖ⁽³⁾α₄)⁻¹r₄ = gᵖ⁽³⁾⁻¹r₄ = gᵖ⁽³⁾⁻¹r₄ = gᵖ⁽³⁾⁴r₄, x ∈ S.

C Proof of Claim

Suppose that we are given a twin DH tuple (e(g, g), e(g, g)ᵖ⁻¹, e(g, g)ᵖ) and the case (g, g)ᵖ⁺ = (g, g)ᵖ⁻¹ (g, g)ᵖ⁺ = (g, g)ᵖ⁻¹, hence the algorithm never occurs: a contradiction. So we have

D Proof of Claim

To reduce to the target collision resistance of an employed hash function family HHam, we construct a PPT target collision finder CF that attacks HHam using A as a subroutine. The construction is shown in Fig.2 (Note that the case Collision is defined in Fig.2).

Note that the view of A in CF is the same as the real view of A until the case Collision occurs and hence the algorithm A works as designed.

To evaluate the probability in Claim 5 we consider the following two cases.

Case 1: the case that Abort (r = r⁺) occurs in B in Phase 1. In this case, the target r⁺ has not been given to A. So A needs to guess r⁺ to cause a collision t = r⁺. Hence:

Pr[Phase 1 ∩ \( \bigvee_{i=1}^{d₂} \neg \text{Overlook}_i \) ∩ Abort] ≤ Pr[Phase 1 ∩ Collision].

Case 2: the case that Abort (r = r⁺) occurs in B in Phase 2. In this case, if, in addition to r = r⁺, it occurred that ψᶜpa = ψᶜpa⁺ (and hence C = C⁺), then it would occur that ψ = ψ⁺. This is because the following two tuples are equal twin DH tuples by the fact that Overlook, never occurs:

\( (e(g, g), e(g, g)ᵖ⁺) = (e(g, g), e(g, g)ᵖ⁻¹, e(g, g)ᵖ⁺) = (e(g, g), e(g, g)ᵖ⁻¹, e(g, g)ᵖ⁺) = (e(g, g), e(g, g)ᵖ⁺) = (e(g, g), e(g, g)ᵖ⁻¹, e(g, g)ᵖ⁺) = (e(g, g), e(g, g)ᵖ⁺) = (e(g, g), e(g, g)ᵖ⁻¹, e(g, g)ᵖ⁺, (C⁺)g, d₁, d₂),

Hence both S ∈ A and ψ = ψ⁺ would occur. This is ruled out in descapsulation query; a contradiction. So we have ψᶜpa ≠ ψᶜpa⁺; that is, a collision:

\( ψᶜpa ≠ ψᶜpa⁺ ∩ \neg H(ψᶜpa) = τ = r⁺ = H(ψᶜpa⁺) \). Therefore, if Overlook, never occurs for each i, then only descapsulation query for which (e(g, g), e(g, g)ᵖ⁻¹, e(g, g)ᵖ⁺, Y, Z₁, Z₂) are certainly twin DH tuples have the chance to cause a collision r = r⁺, as is the case in CF. Hence we have:

Pr[Phase 2 ∩ \( \bigvee_{i=1}^{d₂} \neg \text{Overlook}_i \) ∩ Abort] ≤ Pr[Phase 2 ∩ Collision].
Proof of Claim 6

Taking a sum of both sides of (6) and (7), we get:

\[ \text{Adv}_{\text{CF}, \text{Hfam}}(\lambda) = \frac{1}{\lambda} \sum_{i=1}^{\lambda} \Pr[\text{Adv}_{\text{CF}, \text{Hfam}}(\lambda) = \text{Adv}_{\text{CF}, \text{Hfam}}(\lambda)] \]

This is deduced as follows:

\[ \text{Adv}_{\text{CF}, \text{Hfam}}(\lambda) = \frac{1}{\lambda} \sum_{i=1}^{\lambda} \Pr[\text{Adv}_{\text{CF}, \text{Hfam}}(\lambda) = \text{Adv}_{\text{CF}, \text{Hfam}}(\lambda)] \]

\[ \text{Adv}_{\text{CF}, \text{Hfam}}(\lambda) = \frac{1}{\lambda} \sum_{i=1}^{\lambda} \Pr[\text{Adv}_{\text{CF}, \text{Hfam}}(\lambda) = \text{Adv}_{\text{CF}, \text{Hfam}}(\lambda)] \]

F Proof of Claim 7

A direct calculation with equalities (1) shows:

\[ d_i' = e(C_{i+1}^a, g)^{\beta_i} = e(g, g)^{\beta_i(\alpha_1(\alpha_2+1))} = e(g, g)^{\beta_i(\alpha_2+1)} \]

Hence \( \psi' = (\psi_{\text{CPA}}', d_1', d_2') \) is legitimate and correctly distributed.