Robust synchronization of nonfragile control of complex dynamical network with stochastic coupling and time-varying delays

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1 Introduction

This article is an extension of a paper previously presented in the international conference of ubiquitous and future networks 2018 (ICUFN 2018) [1]. The great works of Watts and Strogatz [2] which focused on investigation of complex networks have witnessed a tremendous attention from many scientific communities because of the theoretical importance and practical implementation of such outcome in areas such as computer networks, social networks, biological networks, communication networks, electric power grid, food webs and transportation networks [3-6, 8]. Complex dynamical networks (CDNs) are large number of interconnected nodes with each node having some defined contents. Majority of these networks display some level of complexities in their overall topology as well as dynamical properties [9]. Amongst the important collective behaviors of CDNs is synchronization problem. This behavior has been investigated by many profound researchers [10, 11]. Synchronization of a network is related to subsystems been represented as nodes in coupled systems in which the various nodes with different initial conditions converge to a common behavioral trajectory. Many control methodologies have been proposed to ensure the solution to synchronization problems, amongst such control methods are pinning control [12], sampled data control [13], sliding mode control [14], impulsive control [15] and so on. The tendencies of sudden changes in network coupling which can emanate from internal and external environmental factors such as unexpected change of working environment, random link failures and repairs on network connectivity result in stochastic behavior in the network coupling. Additionally, the practical implementation of control design might not be precise because of limited information speed, round-off error of numerical computations, aging of system components, analogy to digital conversions (ADC) and Digital to analogy conversion (DAC) [13]. Controller fragility refers to the variation effects on the control parameters as cited in [16]. This problem is addressed in the design of a nonfragile control...
scheme which is presented to address controller gain fluctuation based on the aforementioned shortfalls as presented in [17][18].

Time delays which are ubiquitous in nature and present in CDNs, possess the ability to destroy synchronization performance which can lead to oscillation and instability of the network, hence the need to consider it when addressing synchronization issues of CDNs. It is very important to also note that, there exit time delays in the exchange of information resulting from finite transmission speed in the network, memory effect, limited bandwidth and so on [19]. From the aforementioned discussions, we are motivated in this paper to seek solutions to CDNs synchronization with nonfragile control design scheme, taking into consideration the stochastic coupling nature of the network. The important contributions of this paper are summarized as follows:

1. The problem of CDNs robust synchronization with stochastic coupling and time-varying delays is studied permitting some level of control gain uncertainties.

2. Control scheme with non-fragile characteristics is designed and presented which guarantee the system error synchronization.

3. A suitable Lyapunov Krasovskii functional (LKF) is chosen which applied the extended Jensen’s integral inequality.

4. Given are two numerical examples which indicate the usefulness of our proposed control approach.

2 Model description and Preliminaries

In this paper, standard notations are used. $\mathbb{R}^n$ shows the Euclidean n-dimensional space, $\mathbb{R}^{m \times n}$ denotes the set of all $m \times n$ real matrices. 1 and $0$ represent identity and zero matrices with appropriate dimensions, respectively. $P > 0$ is a real positive symmetric definite matrix. The superscript “$T$” indicates transposition. Also, an asterisk(*) is used to show the symmetric terms and diag(...) represent a diagonal block matrix. All other matrices without given dimensions are considered to be of compatible dimensions. In this paper, the robust synchronization of CDNs which entail the stochastic coupling of N identical nodes with time varying delays is described as follows:

$$
\dot{\tilde{r}}_i(t) = \hat{A}\tilde{r}_i(t) + B f(\tilde{r}_i(t)) + (1 - \delta_1(t))\sum_{j=1}^{N} \tilde{c}_{ij}\Gamma \tilde{r}_j(t) \\
+ \delta_1(t)\sum_{j=1}^{N} \tilde{c}_{ij}\Gamma \tilde{r}_j(t) - \tilde{y}(t)) + \hat{u}(t),
$$

$$
i = 1,2,\ldots,N,
$$

Where, $\tilde{r}_i(t) = (\tilde{r}_{i1}(t), \tilde{r}_{i2}(t), \ldots, \tilde{r}_{IN}(t))^T \in \mathbb{R}^n$ denotes the state vector of the $i^{th}$ node, $\tilde{y}(t) \in \mathbb{R}^n$, $A, B$ are known real constant matrices and they are assumed to be stabilizable, $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ indicate a smooth nonlinear function, $\Gamma \in \mathbb{R}^{nxn}$ is the inner coupling matrix of the nodes and $\hat{C} = (\hat{c}_{ij})_{N \times N}$ represent the outer coupling configuration of the network. If there is a connection from node $i$ to node $j$ ($i \neq j$), then the coupling matrix $\hat{c}_{ij} \neq 0$; otherwise $\hat{c}_{ij} = 0$. Furthermore, the diagonal elements are defined as $\hat{c}_{ii} = -\sum_{j=1,j\neq i}^{N} \hat{c}_{ij}$, $\hat{y}(t)$ represent time-varying delay which is considered to be a differentiable function that satisfies the following conditions:

$$
0 \leq \hat{y}_1(t) \leq \hat{y}_2(t), \quad \hat{y}(t) \leq \bar{\mu}.
$$

$\delta_1(t) \in \mathbb{R}$ denotes a stochastic variable, which is in the form of a Bernoulli distribution sequence defined by

$$
\delta_1(t) = \begin{cases} 
1 & \text{presence of delay in information exchange}, \\
0 & \text{no delay in information exchanges}
\end{cases}
$$

Below indicates the stochastic probability variable $\delta_1(t)$:

$$
Pr[\delta_1(t) = 1] = \delta_1,
$$
$$
Pr[\delta_1(t) = 0] = 1 - \delta_1,
$$

where $\delta_1 \in [0,1]$ is a known constant. Then, the supposed initial condition for $\tilde{r}_i(t)$ is given by $\tilde{r}_i(t) = \tilde{y}_0(t)$, $t \in [-\gamma,0]$ and $i = 1,\ldots,N$.

Assumption 2.1 (20) The continuous vector valued function $f(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is considered which satisfies $f(0) = 0$, hence this sector-bounded condition stands:

$$
[f(x) - f(y) - Z_1(x-y)]^T[f(x) - f(y) - Z_2(x-y)] \leq 0,
$$

given that $Z_1$ and $Z_2$ are constant matrices of appropriate size. From [1] and using kronecker properties, we have

$$
\tilde{r}(t) = (I_N \otimes A)\tilde{r}(t) + (I_N \otimes B)g(\tilde{r}(t)) + (1 - \delta_1(t))(\hat{C} \otimes \Gamma)
$$
$$
\times \tilde{r}(t) + \delta_1(t)(\hat{C} \otimes \Gamma)\tilde{r}(t) - \tilde{y}(t)) + \hat{u}(t)
$$

Where,

$$
\tilde{r}(t) = [\tilde{r}_1^T(t), \tilde{r}_2^T(t), \ldots, \tilde{r}_N^T(t)]^T,
$$
$$
g(\tilde{r}(t)) = [f^T(\tilde{r}_1(t)), f^T(\tilde{r}_2(t)), \ldots, f^T(\tilde{r}_N(t))]^T,
$$

$\hat{u}(t) = [\hat{u}_1^T(t), \hat{u}_2^T(t), \ldots, \hat{u}_N^T(t)]^T$.

Lemma 2.1 (21) Jensen inequality Given a matrix $H = H^T > 0$, of an appropriate dimension and a vector function $\hat{a}(\cdot) : [0, \hat{\gamma}] \rightarrow \mathbb{R}^r$ for a scalar $\hat{\gamma} > 0$, the integration is defined as follows:

$$
\hat{\gamma} \int_0^{\hat{\gamma}} \hat{a}(s)^TH\hat{a}(s)ds \geq \left[\int_0^{\hat{\gamma}} \hat{a}(s)ds\right]^TH\left[\int_0^{\hat{\gamma}} \hat{a}(s)ds\right]^T
$$
Consider the dynamics of an isolated unforced node $s(t)$ to be $\dot{s}(t) = As(t) + B f(s(t))$. $s(t)$ can be taken as an equilibrium point, periodic orbit, or even a chaotic attractor.

Let the $i^{th}$ node error system be given as $e_i(t) = \dot{r}_i(t) - s(t)$. Consequently, error dynamics of CDNs [1] is given by:

$$
\dot{e}_i(t) = A e_i(t) + B g(e_i(t)) + (1 - \delta_1(t)) \sum_{j=1}^{N} \xi_{ij} \Gamma e_j(t)
$$

$$
+ \delta_1(t) \sum_{j=1}^{N} \xi_{ij} \Gamma e_j(t - \gamma(t)) + \dot{a}_i(t)
$$

(6)

Note: $g(e_i(t)) \equiv f(\dot{r}_i(t)) - f(s(t))$.

The designed control scheme to guarantee synchronization of the CDNs is:

$$
\dot{a}_i(t) = (k_i + \sigma(t) \Delta k_i(t)) e_i(t) + k_i e(t - \gamma(t)), \quad i = 1, 2, \ldots, N.
$$

(7)

Consider $k_i, k_x \in \mathbb{R}^{m \times m}$ as the feedback controller gain matrices which are yet to be estimated. Also, $\Delta k_i$ represent the gain fluctuation, where $\Delta k_i(t)$ is known as follows:

$$
\Delta k_i(t) \equiv H_i \Upsilon_i(t) W_i
$$

where $\Upsilon_i(t) \in \mathbb{R}^{kx_i \times l}$, is an unknown time-varying matrix which satisfies the condition: $\Upsilon_i(t)^T \Upsilon_i(t) \leq I$. $H_i$ and $W_i$ are matrices of known parameters. The probability variable $\sigma(t)$ shows controller gains fluctuations. The random occurring fluctuations of the gain obeys the Bernoulli distribution with the following definition

$$
\sigma(t) = \begin{cases} 
1 & \text{Gain fluctuation occur,} \\
0 & \text{Gain fluctuation does not occur}
\end{cases}
$$

and the probability of stochastic parameter $\sigma(t)$ is given as:

$$
Pr(\sigma(t) = 1) = \sigma \quad Pr(\sigma(t) = 0) = 1 - \sigma; \quad \sigma \in [0, 1]
$$

Closed loop of the error dynamics [6] of the CDNs yields:

$$
\dot{e}_j(t) = A e_i(t) + B g(e_i(t)) + (1 - \delta_1(t)) \sum_{j=1}^{N} \xi_{ij} \Gamma e_j(t)
$$

$$
+ \delta_1(t) \sum_{j=1}^{N} \xi_{ij} \Gamma e_j(t - \gamma(t)) + (k_i + \sigma(t) \Delta k_i(t)) e_i(t)
$$

$$
+ k_i e(t - \gamma(t)) = (A + k_i + \sigma(t) \Delta k_i(t)) e_i(t) + B g(e_i(t))
$$

$$
+ (1 - \delta_1(t)) \sum_{j=1}^{N} \xi_{ij} \Gamma e_j(t) + \delta_1(t) \sum_{j=1}^{N} \xi_{ij} \Gamma e_j(t - \gamma(t))
$$

$$
\times e_j(t - \gamma(t)) + k_i e_i(t - \gamma(t))
$$

Remark 1. The Lemma (2.2) is derived from the reciprocal convex combination techniques with Jensen inequality resulting in the less conservativeness of our results. The detailed proof is omitted but can be referred from [22].

**Lemma 2.3 (Schur Complement [23])**: For a given matrix

$$
\Lambda = \begin{bmatrix}
\Lambda_{11} & \Lambda_{12} \\
* & \Lambda_{22}
\end{bmatrix} < 0
$$

any of the inequalities below is equivalent to $\Lambda$ :

1. $\Lambda_{11} < 0$, $\Lambda_{22} - \Lambda_{12}^T \Lambda_{11}^{-1} \Lambda_{12} < 0$

2. $\Lambda_{22} < 0$, $\Lambda_{11} - \Lambda_{12} \Lambda_{12}^T \Lambda_{22}^{-1} \Lambda_{12} < 0$

**Lemma 2.4 [6]**. Suppose $Q = Q^T$, $R$ and $T$ been real appropriate dimension matrices and the function $F(t)$ which satisfies the condition $F^T(t) F(t) < 1$. Accordingly, $Q + R F(t)^T T^T F(t) R^T < 0$ when a given scalar $\varepsilon > 0$ exist. Then,

$$
\begin{bmatrix}
Q & R & \varepsilon T^T \\
* & -\varepsilon I & 0 \\
* & * & -\varepsilon I
\end{bmatrix} < 0.
$$

**Definition 1** The CDNs [1] is synchronized when the condition below holds:

$$
\lim_{t \rightarrow \infty} [\tilde{r}_i(t) - s(t)] = 0.
$$

Consider the dynamics of an isolated unforced node $s(t)$ to be $\dot{s}(t) = As(t) + B f(s(t))$. $s(t)$ can be taken as an equilibrium point, periodic orbit, or even a chaotic attractor.
\[ (A + k + \alpha H \gamma_1 e(t) + (\alpha(t) - \sigma) H \gamma_2 e(t) + B g(e(t)) + (1 - \delta_1) \sum_{j=1}^{N} \zeta_{ij} \Gamma e_j(t) + (\delta_1 - \delta_1(t)) \sum_{j=1}^{N} \zeta_{ij} \Gamma e_j(t - \gamma(t)) + (\delta_1(t) - \delta_1) \]
\[ \times \sum_{j=1}^{N} \zeta_{ij} \Gamma e_j(t - \gamma(t)) + k_e \epsilon(t - \gamma(t)); \quad (i = 1, 2, \ldots, N) \]
\[ \epsilon(t) = (A + \bar{k} + \alpha \bar{H} \bar{Y} \bar{W}) \epsilon(t) + (\sigma(t) - \sigma) \left( \bar{H} \bar{Y} \bar{W} \epsilon(t) + B \bar{g}(\epsilon(t)) + (1 - \delta_1) (\bar{C} \otimes \Gamma) \epsilon(t) + (\delta_1 - \delta_1(t)) \right) \]
\[ \times (\bar{C} \otimes \Gamma) \epsilon(t) + \delta_1(\bar{C} \otimes \Gamma) \epsilon(t) - \int_{t-\gamma(t)}^{t} \epsilon(s) \, ds \]
\[ + \left( \delta_1(t) - \delta_1 \right) (\bar{C} \otimes \Gamma) \epsilon(t) - \int_{t-\gamma(t)}^{t} \epsilon(s) \, ds \]
\[ + \bar{k}_e \epsilon(t) - \int_{t-\gamma(t)}^{t} \epsilon(s) \, ds \]

where,
\[ \bar{A} \equiv I_N \otimes A \]
\[ \bar{k} \equiv diag(k_1, k_2, \ldots, k_N) \]
\[ \bar{H} \equiv diag(H_1, H_2, \ldots, H_N) \]
\[ \bar{Y} \equiv diag(Y_1, Y_2, \ldots, Y_N) \]
\[ \bar{W} \equiv diag(W_1, W_2, \ldots, W_N) \]
\[ \epsilon(t) \equiv [\epsilon_1(t), \epsilon_2(t), \ldots, \epsilon_N(t)]^T \]
\[ G(\epsilon(t)) \equiv [g_1^T(\epsilon_1(t)), g_2^T(\epsilon_2(t)), \ldots, g_N^T(\epsilon_N(t))]^T \]
\[ k_e \equiv I_N \otimes k_e \]
\[ B \equiv I_N \otimes B \]

Theorem 1 Suppose Assumption 1 holds. For given scalars \( \sigma \in [0, 1], \delta_1, \gamma_1, \gamma_2, \) and \( \mu < 1, \) the closed-loop error system (10) is synchronized with controller gains \( k_i, \bar{k}_i \) \((i = 1, 2, \ldots, N)\). If some positive definite matrices \( P, \bar{S}, W_1, W_2, Z \in \mathbb{R}^{N \times N} \) exist and any given matrices \[ \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \]

such that

\[ \Psi = \begin{bmatrix} \hat{S} & 0 \\ 3\hat{S} & 0 \end{bmatrix} \geq 0, \]

Then

\[ \Phi_1 = \begin{bmatrix} \Omega_1 & M_1^T \bar{P} \\ \bar{P} & -2\bar{P} \end{bmatrix} \]

holds, as

\[ \begin{bmatrix} \Phi_1 & N \epsilon W_e \notag \end{bmatrix} \begin{bmatrix} \epsilon W_e \notag \end{bmatrix} < 0, \]

where

\[ \epsilon = (\bar{A} + \bar{k} + \bar{H} \bar{Y} \bar{W}) \epsilon(t) + (\sigma(t) - \sigma) \bar{H} \bar{Y} \bar{W} \epsilon(t) + B \bar{g}(\epsilon(t)) + (1 - \delta_1) \bar{C} \otimes \Gamma \epsilon(t) + (\delta_1 - \delta_1(t)) \bar{C} \otimes \Gamma \epsilon(t) - \int_{t-\gamma(t)}^{t} \epsilon(s) \, ds \]

3 Main results

This section establishes sufficient conditions for the synchronization purposes of the CDN. Additionally, the method for designing the synchronization controllers are presented in terms of LMIs.

\[ \Theta = \gamma_1 \hat{S} + \gamma_2 Z, \quad \hat{A}_1 = -(1 - \hat{\mu}) W_1 - F_1, \]
\[ \hat{\sigma} = \sqrt{\sigma(1 - \sigma)(P \bar{H})^T}, \quad F_1 = I_N \otimes \frac{U_e^T U_e + U_i^T U_i}{2}, \]
\[ F_2 = I_N \otimes \frac{U_e^T U_e + U_i^T U_i}{2} \]
Proof. The following candidate of Lyapunov-Krasovskii functional is considered

\[
\bar{V}(t) = \bar{V}_1(t) + \bar{V}_2(t) + \bar{V}_3(t)
\]

\[
(12)
\]

Where

\[
\bar{V}_1(t) = e^{\tilde{t}(t)}Pe(t)
\]

\[
\bar{V}_2(t) = \int_{t-\bar{\gamma}1}^t e^{\tilde{t}(s)}W_1 e(s) ds + \int_{t+\phi}^t e^{\tilde{t}(s)}W_2 e(s) \times ds d\phi
\]

\[
\bar{V}_3(t) = \int_{t-\bar{\gamma}2}^t \int_{t+\phi}^t e^{\tilde{t}(s)}(\dot{\bar{S}}e(s)) d s d \phi + \int_{t-\bar{\gamma}1}^t \int_{t+\phi}^t e^{\tilde{t}(s)}(Z e(s)) \times Z e(s) d s d \phi
\]

Finding infinitesimal operator \( \mathcal{L} \) on \( \bar{V}(t) \) results in the following:

\[
L \bar{V}(t) = \lim_{\Delta \to 0} \frac{1}{\Delta} [E[V(t+\Delta)] - V(t)]
\]

One should take note of the following expectation:

\[
E[\delta_1(t) - \delta_1] = 0, \quad E[\delta_1(t) - \delta_1]^2 = \delta_1(1 - \delta_1),
\]

\[
E[\sigma(t) - \sigma] = 0, \quad E[(\sigma(t) - \sigma)^2] = \sigma(1 - \sigma).
\]

\( \bar{V}(t) \) is calculated based on the trajectory of error system [10]

\[
E[L \bar{V}_1(t)] = E[2e^{\tilde{t}(t)}P(\hat{\alpha} + \tilde{\bar{\kappa}} + \sigma H \hat{W} + (\hat{C} \otimes \Gamma)) + \tilde{\bar{\kappa}}(t)G(e(t)) - \delta_1(\hat{C} \otimes \Gamma) \int_{t-\bar{\gamma}1}^t e(s) ds]
\]

\[
- \tilde{\bar{\kappa}} \int_{t-\bar{\gamma}1}^t e(s) ds\]

\[
(13)
\]

\[
E[L \bar{V}_2(t)] \leq E[e^{\tilde{t}(t)}(W_1 + \gamma_1 W_2) e(t) - (1 - \tilde{\mu}) \times e^{\tilde{t}(t-\bar{\gamma}1) W_1 e(t-\bar{\gamma}1) - \frac{1}{\bar{\gamma}1}}
\]

\[
\times \left( \int_{t-\bar{\gamma}1}^t e(\theta) d\theta \right)^T W_2 \left( \int_{t-\bar{\gamma}1}^t e(\theta) d\theta \right)
\]

\[
(14)
\]

\[
E[L \bar{V}_3(t)] \leq E[(\gamma_2 - \bar{\gamma}2)e^{\tilde{t}(t)}(\dot{\bar{S}}e(\bar{\delta})) + \gamma_2 e^{\tilde{t}(t)}(z e(t))]
\]

\[
- \frac{1}{\bar{\gamma}2} \int_{t-\bar{\gamma}2}^t e^{\tilde{t}(\bar{\delta})} \dot{\bar{S}}e(\bar{\delta}) \times (1 - \tilde{\mu}) \int_{t-\bar{\gamma}2}^t e^{\tilde{t}(s)}Z e(s) ds
\]

\[
(15)
\]

Based on lemma [2.2] and from [15], the integral component satisfies the following inequality:

\[
- \frac{1}{\bar{\gamma}2} \int_{t-\bar{\gamma}2}^t e^{\tilde{t}(\bar{\delta})} \dot{\bar{S}}e(\bar{\delta}) \times (1 - \tilde{\mu}) \int_{t-\bar{\gamma}2}^t e^{\tilde{t}(s)}Z e(s) ds \leq - \mathcal{Z}(t) \frac{1}{\bar{\gamma}12} \Pi^T \Psi \Pi \mathcal{C}(t)
\]

\[
(16)
\]

Where,

\[
\bar{\gamma}12 = \bar{\gamma}2 - \bar{\gamma}1
\]
The following theorem is presented based on the above.

**Theorem**

The application of Schur complements formula yeilds the following:

\[
\Phi = \begin{bmatrix}
\Omega_1 & M_1^T P & \Omega_2 & 0 \\
* & \Theta - 2P & 0 & 0 \\
* & * & \Theta - 2P & 0 \\
* & * & * & \Theta - 2P
\end{bmatrix}
\]

where, \( \Omega_2 = \Omega_1 + \Omega_2 + \Omega_2^T \), \( \Omega_2 = \sqrt{\sigma(1 - \sigma)M_1^T P} \).

By using the method of congruence transformation with \( \text{diag}(I, I, \ldots, I, P, P, P) \) on \( \Phi \) and utilizing \( P\Theta^{-1}P \geq 2P - \Theta \), ensures \( \Phi < 0 \). Applying simple computations considering Lemma (2.4), let \( \Phi_1 = \begin{bmatrix}
\Omega_1 & M_1^T P & \Omega_2 & 0 \\
* & \Theta - 2P & 0 & 0 \\
* & * & \Theta - 2P & 0 \\
* & * & * & \Theta - 2P
\end{bmatrix}, M_{1*} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \]

\( N^T = \begin{bmatrix}
\sigma (P\bar{\Theta})^T & 0, \ldots, 0 & \sigma (P\bar{\Theta})^T & 0, \ldots, 0
\end{bmatrix} \)

\( W_* = \begin{bmatrix}
\hat{W}_0 \\
0_n, \ldots, 0_n
\end{bmatrix} \).

Then we have

\[
\Phi_1 N \epsilon W_0^T \begin{bmatrix}
\epsilon & -\epsilon I & 0 \\
* & \epsilon & -\epsilon I \\
* & * & \epsilon & -\epsilon I
\end{bmatrix} < 0
\]

Based on Lemma (2.4), if the conclusion \( \Phi < 0 \) is true, then \( \text{E}[LV(t)] < 0 \). Hence, the synchronized error system is asymptotically stable. This completes the proof.

The following theorem is presented based on the above results in addressing the nonfragile control design problem.

**Theorem 2**

Let \( \gamma_1, \delta_1, \gamma_2, \sigma \in [0, 1] \), and \( c \in [0, 1] \), be some given scalars. The given CDNs is synchronized, when some symmetric positive definite matrices \( P = \text{diag}(P_1, P_2, \ldots, P_n), S = \text{diag}(S_1, S_2, \ldots, S_n) \) and any matrices \( Y_1 = \text{diag}(Y_1^1, Y_1^2, \ldots, Y_1^n) \in \mathbb{R}^{n \times n N} \) and \( Y_2 = \text{diag}(Y_2^1, Y_2^2, \ldots, Y_2^n) \in \mathbb{R}^{n \times n N} \) exit with the positive scalar \( \epsilon \), hence \( \epsilon \leq 0 \), where

\[
\begin{bmatrix}
\epsilon P\bar{H} & \epsilon \bar{W}_1^T \\
\bar{W}_0 & \epsilon \bar{Z}^T
\end{bmatrix} < 0
\]

**4 Numerical Simulations**

The objective of this section is to exhibit the correctness of our synchronization schemes in Section (3).

**Example 1.** The CDNs (1) is considered which comprises of five nodes with each node been two dimensional, such that \( N = 5 \) and \( n = 2 \). The other parameters involved are:

\[
\begin{bmatrix}
0.7 & 0.2 \\
0.5 & 1.0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 \\
0 & 0 & -1 & 1 & 0 \\
1 & 0 & 0 & 0 & -1
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.5 \text{l}_{11} \\
1.50 & -0.82 \\
5.15 & 2.50
\end{bmatrix}
\]

The controller gain fluctuations made to satisfy \( \Delta k_1(t) \) are defined as

\[
\Delta k_1 = 0.5 \text{l}_{11}, H_2 = 0.6 \text{l}_{11}, H_3 = 0.7 \text{l}_{11}, H_4 = 0.8 \text{l}_{11}, H_5 = 0.9 \text{l}_{11}, W_1 = 0.6 \text{l}_{11}, W_2 = 0.5 \text{l}_{11}, W_3 = 0.4 \text{l}_{11}, W_4 = 0.3 \text{l}_{11}, W_5 = 0.2 \text{l}_{11}
\]
Let $f(\tilde{r}(t))$ be represented as

$$f(\tilde{r}(t)) = \begin{bmatrix} 0.1 \tilde{r}_1 - \tanh(0.1 \tilde{r}_1) \\ 0.1 \tilde{r}_2 \end{bmatrix}. $$

The above $f(\tilde{r}(t))$, satisfies the sector-bounded condition in assumption (2.1) with

$$ Z_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}. $$

Considering this example, we set $\sigma = 0.5, \delta_1 = 0.5, \tilde{\mu} = 0.2, \gamma_1 = 0, \gamma_2 = 0.5$. From MATLAB LMI toolbox, Theorem (2) is verified, where feasible solution of the LMIs in Theorem (2) is obtained. The controller gain matrices obtained are:

$$ K_1 = \begin{bmatrix} -1.3647 & -0.3460 \\ -0.1196 & -1.5990 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -1.3121 & -0.3079 \\ -0.0990 & -1.4745 \end{bmatrix}, \quad K_3 = \begin{bmatrix} -1.2755 & -0.3126 \\ -0.0893 & -1.3825 \end{bmatrix}, \quad K_4 = \begin{bmatrix} -1.2436 & -0.3139 \\ -0.0836 & -1.3191 \end{bmatrix}, \quad K_5 = \begin{bmatrix} -1.2226 & -0.3139 \\ -0.0780 & -1.2753 \end{bmatrix} $$

$$ K_{1\tau} = \begin{bmatrix} -0.2735 & -0.1907 \\ -0.0188 & -0.1500 \end{bmatrix}, \quad K_{2\tau} = \begin{bmatrix} -0.3101 & -0.1871 \\ -0.0325 & -0.2592 \end{bmatrix}, \quad K_{3\tau} = \begin{bmatrix} -0.3163 & -0.1580 \\ -0.0335 & -0.3072 \end{bmatrix}, \quad K_{4\tau} = \begin{bmatrix} -0.3192 & -0.1247 \\ -0.0292 & -0.3276 \end{bmatrix}, \quad K_{5\tau} = \begin{bmatrix} -0.3251 & -0.11251 \end{bmatrix} $$

Using the following initial conditions, $\tilde{r}_1(0) = [3, -1]^T$, $\tilde{r}_2(0) = [0, 1]^T$, $\tilde{r}_3(0) = [-6, 2]^T$, $\tilde{r}_4(0) = [3, -2]^T$, $\tilde{r}_5(0) = [-1, 1]^T$, and $s(0) = [2, 3]^T$. The error trajectories is shown in figure (1) without control input. The control inputs and error system trajectories with control inputs are shown in figures (2) and (3) respectively.
The parameters considered are: the inner coupling \( \Gamma \) and network topology \( \tilde{C} \) matrices are given as
\[
\Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}
\]
The parameters considered are: \( \gamma_1 = 0.01, \gamma(t) = 0.4 + 0.01 \sin(10t) \), \( \gamma_2 = 0.41, \bar{\mu} = 0.1, \sigma = 0 \). MATLAB LMI toolbox is used on Theorem (1). The obtained controller gain matrices are
\[
k_1 = \begin{bmatrix} -1.6520 & 0.3835 & 0.0811 \\ 0.6663 & -0.9311 & 0.7839 \\ 0.0602 & 0.4338 & -1.3760 \end{bmatrix},
k_2 = \begin{bmatrix} -2.6121 & 0.3850 & 0.2110 \\ 0.5820 & -1.9887 & 1.1084 \\ 0.0163 & 0.5489 & -1.1378 \end{bmatrix},
k_3 = \begin{bmatrix} -0.7934 & 0.3540 & 0.0905 \\ 0.7406 & 0.6739 & 0.2685 \\ 0.0820 & 0.2063 & -0.5514 \end{bmatrix},
k_{r1} = \begin{bmatrix} -0.1015 & 0.0165 & -0.0144 \\ -0.0051 & -0.5132 & 0.3061 \\ -0.0022 & 0.1375 & -0.1897 \end{bmatrix},
k_{r2} = \begin{bmatrix} -0.0967 & -0.0076 & -0.0082 \\ -0.0175 & -0.3802 & -0.0082 \\ -0.0007 & -0.0081 & -0.1502 \end{bmatrix},
k_{r3} = \begin{bmatrix} -0.0810 & 0.0670 & -0.0145 \\ -0.0070 & -1.2975 & 1.2294 \\ -0.0177 & 0.5618 & -0.0825 \end{bmatrix}
\]
Assume the following initial conditions are considered for the system: \( s(0) = [0, -1, 1]^T, \tilde{r}_1(0) = [1, -2, 8]^T, \tilde{r}_2(0) = [4, -6, 4]^T, \) and \( \tilde{r}_3(0) = [1, -1, 7]^T \). The simulation result given in figure (4) indicates state error trajectories without control input, whereas that of figures (5) and (6) depict the synchronized closed-loop error system and the control input signals respectively.

### 5 Conclusion

This paper shows how an appropriate Lyapunov Krasovskii functional is used to address a non-fragile synchronization control problem for a stochastic coupling complex dynamical networks with time-varying delays and control gain perturbations. The application of extended version of Jensen’s inequality ensured the LMI to be feasible. Finally, numerical examples with simulations are shown to illustrate the validity and applicability of our proposed control scheme.

### Conflict of Interest
The authors declare no conflict of interest.

### Acknowledgment
The authors would like to express their profound gratitude to Professor Zhong Shouming of UESTC for his immense support and contributions.

### References


